Exercises on
Ship and Offshore Hydromechanics

(in progress of formation)

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Contents:

Exercise 1. Regular Waves in a Towing Tank.

A towing tank has a length of 150.00 m, a width of 5.00 m and a wave maker in one end that generates long-crested regular waves. Assume in this tank a generated regular deep water wave with amplitude $\zeta_a = 0.200\ \text{m}$ and wave period $T = 2.0\ \text{s}$.
The velocity potential of the wave is given by:

$$\Phi_w = \frac{\zeta_a \cdot g}{\omega} \cdot e^{k \cdot z} \cdot \sin\left(k \cdot x - \omega \cdot t\right) \qquad \text{with: } \rho = 1000\ \text{kg/m}^3 \text{ and } g = 9.81\ \text{m/s}^2.$$

Exercise:
Determine:
1. The circular wave frequency, ω, the wave number, k, and the wave length, λ.
2. The maximum fluid particle velocities $u_{\max}$ and $w_{\max}$ in x and z directions in the fluid in the tank.
3. The path of a fluid particle in the surface of the wave.
4. The path of a fluid particle at 0.500 meter (average) below the still water level.
5. The maximum pressure at 0.500 meter below the still water level.
6. The energy in the waves per unit surface area, E/A.
7. The phase velocity, c, and the group velocity, c_g, of these waves.
8. The time needed by this generated wave train to reach the other end of the tank.
9. The time between passing an observer along the tank side of two successive wave crests.
10. The phase of the wave elevation 2.500 meter closer to the wave maker relative to the position of an observer.
11. Suppose now this regular wave being a shallow water wave in a tank with a water depth of 2.000 meter, described by:

$$\Phi_w = \frac{\zeta_a \cdot g}{\omega} \cdot \frac{\cosh\left\{k \cdot (h+z)\right\}}{\cosh\left\{k \cdot h\right\}} \cdot \sin\left(k \cdot x - \omega \cdot t\right)$$

Determine the maximum fluid particle displacements $x_{\max}$ and $z_{\max}$ as well as velocities $u_{\max}$ and $w_{\max}$ in x and z directions at the surface of the fluid as well as at the bottom of the tank.

Solution:
1. $\omega = 3.142$ rad/s, $k = 1.005\ \text{m}^{-1}$ and $\lambda = 6.252$ m.
2. $u_{\max} = w_{\max} = 0.628$ m/s.
3. Circular motion with a radius of 0.200 m.
4. Circular motion with a radius of 0.121 m.
5. $p_{\max} = 6020\ \text{N/m}^2$.
6. $E/A = 196$ N/m.
7. $c = 3.126$ m/s and $c_g = 1.563$ m/s.
8. $t = 96.0$ s.
9. $t = T = 2.0$ s.
10. $\varepsilon = -144.0^0$.
11. $x_{\max}(z=0) =$ 0.207m and $u_{\max}(z=0) =$ 0.651 m/s.
$z_{\max}(z=0) =$ 0.200 m and $w_{\max}(z=0) =$ 0.628 m/s.
$x_{\max}(z=-2.000) =$ 0.055 m and $u_{\max}(z=-2.000) =$ 0.171 m/s
$z_{\max}(z=-2.000) =$ 0.000 m and $w_{\max}(z=-2.000) =$ 0.000 m/s.

Exercise 2. Fluid Motions in a Rectangular Tank.

Consider a fixed rectangular tank with a breadth $2b$ partly filled with water until a level h. The fluid is moving in the (y,z) plane with the y axis in the still water plane and the z axis vertically upward.
The 2-D velocity potential of the wave in this tank is given by:

$$\Phi = A \cdot \cosh\{k \cdot (h+z)\} \cdot \cos(k \cdot y) \cdot \cos(\boldsymbol{w} \cdot t)$$

Exercise:

1. Show that this velocity potential satisfies the Laplace equation.
2. Show the boundary condition at the bottom of the tank.
3. Which values of the wave number, k, are possible to satisfy the boundary conditions on the two side walls of the tank?
4. Show from the free surface condition that fluid motions are possible for the natural periods of the tank, T_n, only and that these periods are given by:
$$T_n = \frac{2 \cdot \boldsymbol{p}}{\sqrt{n \cdot \frac{\boldsymbol{p} \cdot g}{b} \cdot \tanh\left(n \cdot \frac{\boldsymbol{p} \cdot h}{b}\right)}} \qquad \text{for}: n = 1,2,3,...$$
5. Derive an approximating formula for T_n when $h/b \rightarrow 0$.
6. Describe the fluid particle motion at the free surface as a function of location and time.

Solution:

1. $\dfrac{\partial^2 \Phi}{\partial y^2} + \dfrac{\partial^2 \Phi}{\partial z^2} = 0$
2. $\left.\dfrac{\partial \Phi}{\partial z}\right|_{z=-h} = 0$
3. $k = n \cdot \dfrac{\boldsymbol{p}}{b}$ for: $n = 1,2,3,...$
4. …
5. $T = \dfrac{2 \cdot b}{\sqrt{g \cdot h}}$
6. $\boldsymbol{z} = \boldsymbol{z}_a \cdot \sin(\boldsymbol{w} \cdot t)$ with: $\boldsymbol{z}_a = \dfrac{\boldsymbol{w}}{g} \cdot A \cdot \cosh(k \cdot h) \cdot \cos(k \cdot y)$

Exercise 3. Wave Spectra Characteristics.

A simplified wave spectrum is given by:

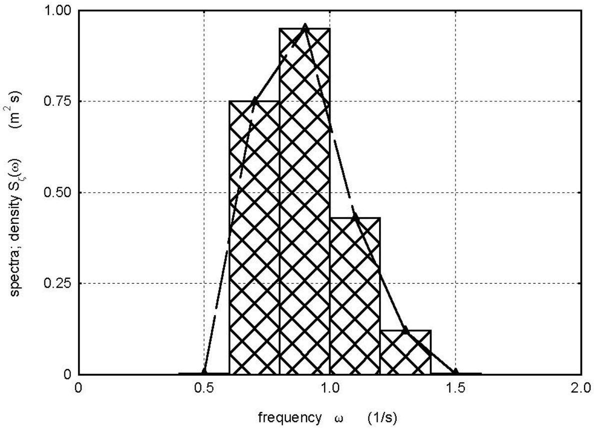

Figure 1: Sketch of a wave spectrum

w (s^{-1})	0.5	0.7	0.9	1.1	1.3	1.5
$S_z(w)$ (m^2s)	0.00	0.75	0.95	0.43	0.12	0.00

<u>Exercise:</u>

1. Calculate (by using the trapezoid rule) the significant wave height, $H_{1/3}$, and the mean wave periods T_1 and T_2. Give a physical explanation of each of these phenomena.
2. Determine the probability, P, of exceeding a wave height of 4.0 meter in this wave spectrum by using the Rayleigh probability density function.
3. Determine also the number of times per hour that this wave height will be exceeded.
4. Explain the term m_0 in the Rayleigh probability density function.
   Which restriction should be kept in mind when using this probability density function?
5. How can this m_0 be determined (2 methods) when a wave elevation record (during for instance half an hour) is available?
6. What is the probability that the significant wave height, $H_{1/3}$, will be exceeded?
7. Transform the wave spectrum on basis of circular wave frequencies, $S_z(w)$, as given above, to a wave spectrum, $S_z(f)$, on basis of frequencies in Herz ($f = 1/T$) and give a sketch of this spectrum.
8. What is the disadvantage of the use of T_2 when analyzing a measured wave spectrum?

<u>Solution:</u>

1. $H_{1/3} = 2.68\text{ m}$, $T_1 = 7.0\text{ s}$ and $T_2 = 6.9\text{ s}$.
2. $P = 0.012$.
3. $N = 6 \text{ times per hour}$.
4. Measure for the amount of energy of the considered variable.
5. A histogram of wave heights or by spectral analysis.
6. $P = 0.135$.
7. …
8. Uncertain tail of spectrum can have a large influence.

Exercise 4. Spectrum of a Combined Sea and Swell.

A fully developed sea and swell are defined by:

$$H_{1/3\ \text{sea}} = 3.0\ \text{m} \qquad H_{1/3\ \text{swell}} = 4.0\ \text{m}$$
$$T_{1\ \text{sea}} = 6.0\ \text{s} \qquad \text{and} \qquad T_{1\ \text{swell}} = 12.0\ \text{s}$$
$$T_{2\ \text{sea}} = 5.5\ \text{s} \qquad T_{2\ \text{swell}} = 11.0\ \text{s}$$

Exercise:

1. Sketch the spectra of this sea and swell and of the combined sea and swell.
2. Calculate the characteristics $H_{1/3}$, T_1 and T_2 of the combined sea and swell.

Solution:

1. Sketch of spectra:

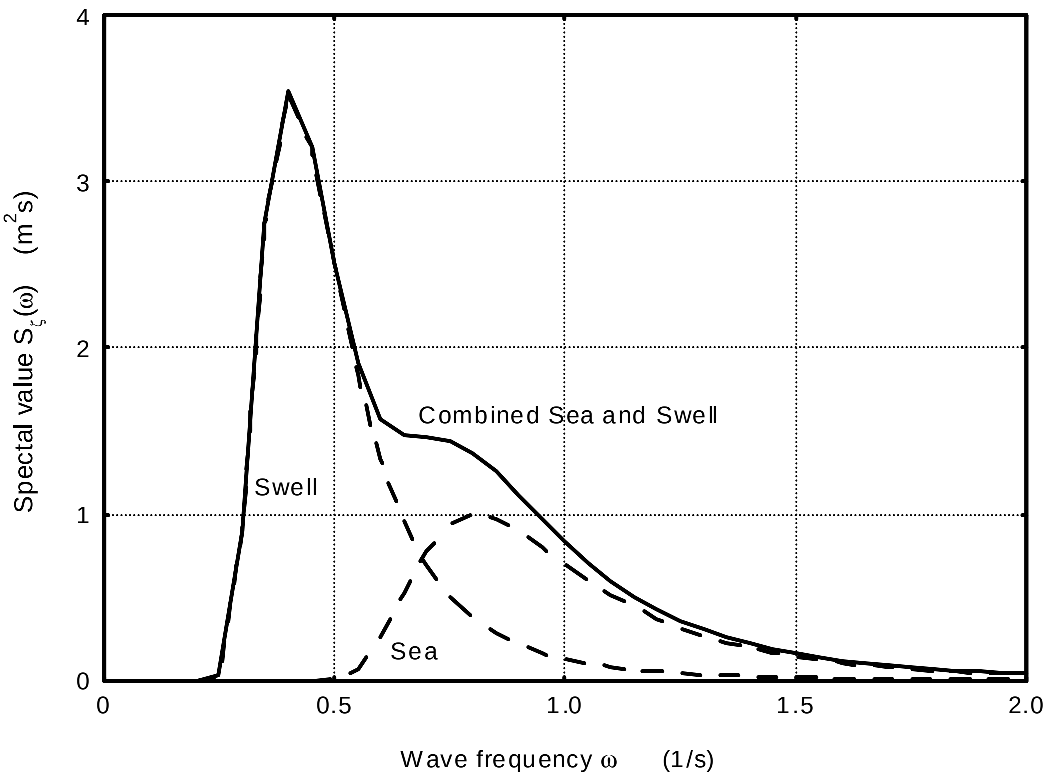

Figure 2: Sketch of sea and swell wave spectra

2. Characteristics:

$$H_{1/3\ \text{combined}} = 5.0\ \text{m}$$
$$T_{1\ \text{combined}} = 8.8\ \text{s}$$
$$T_{2\ \text{combined}} = 7.6\ \text{s}$$

Exercise 5. Radii of Inertia of the Ship's Solid Mass.

When the solid mass distribution of a ship is approximated by the mass of a homogeneous long slender rectangular beam with length L, breadth B and depth H, the three radii of inertia of the ship's solid mass can be approximated by:

$$k_{xx} = 0.30 \cdot B$$
$$k_{yy} = 0.29 \cdot L$$
$$k_{zz} = 0.30 \cdot L$$

Exercise:

1. Demonstrate the validity of these three propositions.
2. Discuss the considerations when obtaining the actual values for ships.

Exercise 6. Estimation of Natural Periods of Ships.

The following dimensions of a ship are known:

Length	L	200.00	m
Breadth	B	30.00	m
Draught	d	10.00	m
Block coefficient	C_B	0.60	
Water plane area coefficient	C_W	0.75	
Initial metacentric height	$\overline{GM}$	1.00	m

Exercise:

Estimate the natural periods for the heave, roll and pitch motions of the ship.
Required unknown parameters may be estimated.,

Solution:

$T_{0z} \approx 7.9 \text{ s}$

$T_{0f} \approx 24.0 \text{ s}$

$T_{0q} \approx 9.9 \text{ s}$

Exercise 7. Free Roll Decay Tests.

A ship - with mass of displacement $m = 4000\,\text{ton}$, length $L = 90\,\text{m}$ and beam $B = 15\,\text{m}$ - has a transverse initial metacentric height $\overline{GM} = 1.50\,\text{m}$. The inertia of the ship's solid mass moment of inertia for roll can be estimated by $k_{xx} = 0.35 \cdot B$.

Free roll decay tests have been carried out with a 1:50 model of this ship. The measured natural rolling period was $T_{0\,\text{model}} = 1.414\,\text{s}$ and the successive roll amplitudes, as the motion of the model was allowed to die down, were: ......, +6.8, -5.6, +4.5, -3.7, ... degrees.

The uncoupled roll equation of motion during the free decay tests is given by:

$$\left(I_{xx} + a_{\phi\phi}\right) \cdot \ddot{\phi} + b_{\phi\phi} \cdot \dot{\phi} + c_{\phi\phi} \cdot \phi = 0$$

Exercise:

Deduce all (hydro)mechanical coefficients - including dimensions - in the ship's uncoupled equation of motion for roll.

Note: Use $g = 9.81\,\text{ms}^{-2}$.

Solution:

$$I_{xx} = 110250\,\text{kNms}^2$$

$$a_{\phi\phi} = 38995\,\text{kNms}^2$$

$$b_{\phi\phi} = 12334\,\text{kNms}$$

$$c_{\phi\phi} = 58860\,\text{kNm}$$

Exercise 8. Scaling Data from Model Scale to Full Scale.

A ship model is scaled by scale $1:\boldsymbol{a}$.

Consider the coupled equations of motion for heave and pitch of this ship model during experiments in regular head waves with forward speed:

$$\boldsymbol{z} = \boldsymbol{z}_a \cdot \cos(\boldsymbol{w} \cdot t)$$

$$(m + a_{zz}) \cdot \ddot{z} + b_{zz} \cdot \dot{z} + c_{zz} \cdot z + d_{zq} \cdot \ddot{\boldsymbol{q}} + e_{zq} \cdot \dot{\boldsymbol{q}} + f_{zq} \cdot \boldsymbol{q} = F_a \cdot \cos(\boldsymbol{w} \cdot t + \boldsymbol{e}_{Fz})$$
$$(I_{yy} + a_{qq}) \cdot \ddot{\boldsymbol{q}} + b_{qq} \cdot \dot{\boldsymbol{q}} + c_{qq} \cdot \boldsymbol{q} + d_{qz} \cdot \ddot{z} + e_{qz} \cdot \dot{z} + f_{qz} \cdot z = M_a \cdot \cos(\boldsymbol{w} \cdot t + \boldsymbol{e}_{Mz})$$

$$z = z_a \cdot \cos(\boldsymbol{w} \cdot t + \boldsymbol{e}_{zz})$$
$$\boldsymbol{q} = \boldsymbol{q}_a \cdot \cos(\boldsymbol{w} \cdot t + \boldsymbol{e}_{qz})$$

These experiments have been followed by another set of experiments in irregular waves. The wave and ship motions have been measured and the data have been stored for further use as wave and motion spectra.

Exercise:

Determine the dimensions (in N, m and s) and the scale factors ($1:\boldsymbol{a}^x$) of:

1. Time t and natural periods T_{0z} and T_{0q}
2. Forward speed V
3. Frequencies $\boldsymbol{w}$ and $\boldsymbol{w}_e$
4. Wave surface elevations $\boldsymbol{z}$ and $\boldsymbol{z}_a$
5. Wave length $\boldsymbol{l}$
6. Displacements z and z_a
7. Velocity $\dot{z}$
8. Acceleration $\ddot{z}$
9. Angular displacements $\boldsymbol{q}$ and $\boldsymbol{q}_a$
10. Angular velocity $\dot{\boldsymbol{q}}$
11. Angular acceleration $\ddot{\boldsymbol{q}}$
12. Wave forces F and F_a
13. Wave moments M and M_a
14. Phase lags $\boldsymbol{e}_{zz}$, $\boldsymbol{e}_{qz}$, $\boldsymbol{e}_{Fz}$ and $\boldsymbol{e}_{Mz}$
15. Solid mass m
16. Volume of displacement ∇
17. Solid mass moment of inertia I_{xx}
18. Radius of inertia k_{yy}
19. Hydrodynamic coefficient a_{zz}
20. Hydrodynamic coefficient b_{zz}
21. Hydrostatic coefficient c_{zz}
22. Hydrodynamic coefficient d_{zq}

23. Hydrodynamic coefficient $e_{z\theta}$
24. Hydrostatic coefficient $f_{z\theta}$
25. Hydrodynamic coefficient $a_{\theta\theta}$
26. Hydrodynamic coefficient $b_{\theta\theta}$
27. Hydrostatic coefficient $c_{\theta\theta}$
28. Hydrodynamic coefficient d_{θ}
29. Hydrodynamic coefficient e_{θ}
30. Hydrostatic coefficient f_{θ}
31. Mean added resistance in regular waves R_{aw}
32. Spectral value of wave elevation $S_\zeta(\omega)$
33. Spectral value of heave displacement $S_z(\omega_e)$
34. Spectral value of heave velocity $S_{\dot{z}}(\omega_e)$
35. Spectral value of heave acceleration $S_{\ddot{z}}(\omega_e)$
36. Spectral value of pitch displacement $S_\theta(\omega_e)$
37. Spectral value of pitch velocity $S_{\dot{\theta}}(\omega_e)$
38. Spectral value of pitch acceleration $S_{\ddot{\theta}}(\omega_e)$

Solution:

1. Time t and natural periods T_{0z} and $T_{0\theta}$	s	$1:\alpha^{0.5}$
2. Forward speed V	ms^{-1}	$1:\alpha^{0.5}$
3. Frequencies ω and ω_e	s^{-1}	$1:\alpha^{-0.5}$
4. Wave surface elevations ζ and ζ_a	m	$1:\alpha$
5. Wave length λ	m	$1:\alpha$
6. Displacements z and z_a	m	$1:\alpha$
7. Velocity $\dot{z}$	ms^{-1}	$1:\alpha^{0.5}$
8. Acceleration $\ddot{z}$	ms^{-2}	$1:1$
9. Angular displacements θ and θ_a	-	$1:1$
10. Angular velocity $\dot{\theta}$	s^{-1}	$1:\alpha^{-0.5}$
11. Angular acceleration $\ddot{\theta}$	s^{-2}	$1:\alpha^{-1}$
12. Wave forces F and F_a	N	$1:\alpha^{3}$
13. Wave moments M and M_a	Nm	$1:\alpha^{4}$
14. Phase lags $\varepsilon_{z\zeta}$, $\varepsilon_{\theta\zeta}$, $\varepsilon_{F\zeta}$ and $\varepsilon_{M\zeta}$	-	$1:1$
15. Solid mass m	Ns^{2}m^{-1}	$1:\alpha^{3}$
16. Volume of displacement ∇	m^3	$1:\alpha^{3}$
17. Solid mass moment of inertia I_{xx}	Ns^{2}m	$1:\alpha^{5}$
18. Radius of inertia k_{yy}	m	$1:\alpha$
19. Hydrodynamic coefficient a_{zz}	Ns^{2}m^{-1}	$1:\alpha^{3}$
20. Hydrodynamic coefficient b_{zz}	Nsm^{-1}	$1:\alpha^{2.5}$
21. Hydrostatic coefficient c_{zz}	Nm^{-1}	$1:\alpha^{2}$

22. Hydrodynamic coefficient $d_{z\theta}$	Ns^2	$1:\alpha^4$
23. Hydrodynamic coefficient $e_{z\theta}$	Ns	$1:\alpha^{3.5}$
24. Hydrostatic coefficient $f_{z\theta}$	N	$1:\alpha^3$
25. Hydrodynamic coefficient $a_{\theta\theta}$	Ns^{2}m	$1:\alpha^5$
26. Hydrodynamic coefficient $b_{\theta\theta}$	Nsm	$1:\alpha^{4.5}$
27. Hydrostatic coefficient $c_{\theta\theta}$	Nm	$1:\alpha^4$
28. Hydrodynamic coefficient $d_{\theta z}$	Ns^2	$1:\alpha^4$
29. Hydrodynamic coefficient $e_{\theta z}$	Ns	$1:\alpha^{3.5}$
30. Hydrostatic coefficient $f_{\theta z}$	N	$1:\alpha^3$
31. Mean added resistance in regular waves R_{aw}	N	$1:\alpha^3$
32. Spectral value of wave elevation $S_{\zeta}(\omega)$	m^{2}s	$1:\alpha^{2.5}$
33. Spectral value of heave displacement $S_{z}(\omega_e)$	m^{2}s	$1:\alpha^{2.5}$
34. Spectral value of heave velocity $S_{\dot{z}}(\omega_e)$	m^{2}s^{-1}	$1:\alpha^{1.5}$
35. Spectral value of heave acceleration $S_{\ddot{z}}(\omega_e)$	m^{2}s^{-3}	$1:\alpha^{0.5}$
36. Spectral value of pitch displacement $S_{\theta}(\omega_e)$	s	$1:\alpha^{0.5}$
37. Spectral value of pitch velocity $S_{\dot{\theta}}(\omega_e)$	s^{-1}	$1:\alpha^{-0.5}$
38. Spectral value of pitch acceleration $S_{\ddot{\theta}}(\omega_e)$	s^{-3}	$1:\alpha^{-1.5}$

Exercise 9. Equations of Motion of a Pontoon.

A laden rectangular pontoon (with a length of 40.00 m, a breadth of 7.50 m, a draught of 1.25 m, a depth of 2.50 m and the center of gravity at 3.00 m above the bottom) floats at even keel condition in seawater.
The equations of motion for six degrees of freedom of this pontoon, at zero forward speed (V = 0.0 kn) in irregular bow waves ($\mu = 150^0$), are given by:

$$\sum_{j=1}^{6} \left\{ \left(M_{i,j} + a_{i,j} \right) \cdot \ddot{x}_j + b_{i,j} \cdot \dot{x}_j + c_{i,j} \cdot x_j \right\} = F_{a_i} \cdot \cos\left(\omega_i t + \varepsilon_{Fz_i} \right) \qquad i = 1,...6$$

Exercise:
Determine in these equations:

- the potential coefficients which have to be zero.
- the spring terms which have to be zero.

Mark these zero values with "0" and the non-zero values with "X" in the tables below and explain why some coefficients are zero.

Surge:

i,j	$a_{i,j}$	$b_{i,j}$	$c_{i,j}$
1,1			
1,2			
1,3			
1,4			
1,5			
1,6			

Sway:

i,j	$a_{i,j}$	$b_{i,j}$	$c_{i,j}$
2,1			
2,2			
2,3			
2,4			
2,5			
2,6			

Heave:

i,j	$a_{i,j}$	$b_{i,j}$	$c_{i,j}$
3,1			
3,2			
3,3			
3,4			
3,5			
3,6			

Roll:

i,j	$a_{i,j}$	$b_{i,j}$	$c_{i,j}$
4,1			
4,2			
4,3			
4,4			
4,5			
4,6			

Pitch:

i,j	$a_{i,j}$	$b_{i,j}$	$c_{i,j}$
5,1			
5,2			
5,3			
5,4			
5,5			
5,6			

Yaw

i,j	$a_{i,j}$	$b_{i,j}$	$c_{i,j}$
6,1			
6,2			
6,3			
6,4			
6,5			
6,6			

Solution:

Ask yourself, when doing this exercise, questions like:

- Is there any symmetry of the body?
- How will potential coefficients being calculated or measured? Or, in other words, what are the environmental conditions?
- Also, ask yourself questions like, for instance: When exerting a vertical force on G in still water, will this cause a surging, a swaying, a rolling, a pitching or a yawing pontoon?

Exercise 10. Wave Spectra Transformation.

The spectrum of an irregular long-crested wave system, as measured at a fixed point, is given by:

ω (rad/s)	0.3	0.4	0.5	0.6
$S_\zeta(\omega)$ (m^2s)	0.30	1.90	4.30	3.80

A ship heads into this wave system at 20 knots (1 knot = 0.5144 m/s) and in a direction such that the velocity vectors for ship and waves incline at 120^0.

Exercise:
Calculate the wave spectrum $S_\zeta(\omega_e)$, as a wave probe moving forward with the speed of the ship would measure it.

Solution:

ω_e (rad/s)	0.347	0.484	0.631	0.789
$S_\zeta(\omega_e)$ (m^2s)	0.228	1.223	2.821	2.332

Exercise 11. Ship Motion Trial.

A ship motion trial has been carried out in a long-crested irregular wave system. The spectrum of the wave system as measured by a wave buoy at a stationary point is defined in the following table:

w (rad/s)	0.3	0.4	0.5	0.6	0.7	0.8
$S_z(w)$ (m^2s)	0.30	1.90	3.23	2.85	2.10	1.40

The heave energy spectrum, obtained by accelerometers on the ship when sailing at 12 knots (1 knot = 0.5144 m/s) on a course of 150^0 relative to the wave direction, is defined as follows:

w_e (rad/s)	0.4	0.5	0.6	0.7	0.8	0.9
$S_z(w_e)$ (m^2s)	0.576	1.624	1.663	0.756	0.149	0.032

Exercise:

Derive the transfer function for heave, $z_a / \boldsymbol{z}_a$, over the given range of frequencies of encounter, w_e.

Solution:

w_e (rad/s)	0.4	0.5	0.6	0.7	0.8	0.9
$z_a / \boldsymbol{z}_a$ (-)	1.03	1.68	0.93	0.62	0.29	0.15

Exercise 12. Superposition of Motions and Spectra.

A ship, with a length (L) of 100.00 meter, sails with a forward speed of 10.0 knots in bow waves ($m = 150^0$) in deep water. A part of the by the strip-theory program SEAWAY of Journée computed non-dimensional amplitude characteristics and the phase characteristics of the motions of the center of gravity G of the ship are given below a function of the wave length over ship length ratio WL/SL (l / L).

FREQUENCY CHARACTERISTICS OF CoG MOTIONS **FORWARD SPEED = 10.00 kn**
~~~~~~ **WAVE DIRECTION = +150 deg off stern**

	...SURGE...		...SWAY....		...HEAVE...		...ROLL....		...PITCH...		....YAW....	
WL/SL	AMPL	PHASE	AMPL	PHASE	AMPL	PHASE	AMPL	PHASE	AMPL	PHASE	AMPL	PHASE
(-)	(-)	(deg)	(-)	(deg)	(-)	(deg)	(-)	(deg)	(-)	(deg)	(-)	(deg)
1.500	0.269	95.6	0.178	262.2	0.681	349.1	0.623	106.5	0.771	249.3	0.203	0.2
1.400	0.240	96.7	0.166	260.9	0.611	347.7	0.414	102.4	0.744	244.9	0.189	0.2
1.300	0.208	98.1	0.149	258.8	0.529	344.4	0.267	100.2	0.708	238.6	0.173	0.3
1.200	0.173	100.1	0.128	255.6	0.428	341.0	0.160	100.1	0.644	229.2	0.155	0.5
1.100	0.134	102.9	0.102	250.9	0.300	342.9	0.083	104.4	0.529	217.7	0.135	0.7
1.000	0.092	108.0	0.073	242.9	0.205	347.8	0.031	127.0	0.376	203.4	0.112	1.0
0.900	0.049	120.9	0.044	225.5	0.134	5.4	0.023	212.6	0.209	193.7	0.086	1.2
0.800	0.021	185.4	0.026	176.7	0.107	25.3	0.039	239.1	0.084	197.3	0.057	1.3
0.700	0.041	250.9	0.033	122.8	0.081	38.1	0.044	244.5	0.034	244.9	0.028	1.0
0.600	0.056	265.7	0.040	103.1	0.044	50.7	0.035	245.9	0.028	284.8	0.003	343.7
0.500	0.040	274.7	0.026	93.4	0.013	109.4	0.015	245.5	0.014	302.4	0.012	190.8

A part of the energy spectrum of the irregular waves is given below in the column with l / L (= WL/SL) ratios, the not-filled column with w's and the column with $S_z(w)$ values.

		SPECTRAL
WL/SL	OMEGA	VALUE
(-)	(rad/s)	(m2s)
1.500	..	3.420
1.400	..	3.158
1.300	..	2.862
1.200	..	2.538
1.100	..	2.198
1.000	..	1.853
0.900	..	1.513
0.800	..	1.190
0.700	..	0.895
0.600	..	0.634
0.500	..	0.417

The co-ordinates of a point B at the port side of the ship's bridge (in an axes system with the origin at G) are given by:

$$x_B = -40.00 \text{ m} \qquad y_B = +10.00 \text{ m} \qquad z_B = +15.00 \text{ m}$$

<u>Exercise:</u>

Determine the spectral values of the motions of this point B, based on the wave frequency **and** on the frequency of encounter:

- For the horizontal longitudinal absolute displacement x at $l / L = \ldots$
- For the horizontal lateral absolute displacement y at $l / L = \ldots$
- For the vertical absolute displacement z at $l / L = \ldots$
- For the vertical relative displacement s relative to the undisturbed wave surface elevation at $l / L = \ldots$

<u>Remarks</u>:

- 1 knot = 0.5144 m/s and g = 9.806 m/s^2.
- Answers have to be given with 3 decimals.
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Solution:

The energy spectra of the waves and the four motions of point B based on the wave frequency are:

WAVE FREQ (r/s)	WAVE (m2s)	X-abs (m2s)	Y-abs (m2s)	Z-abs (m2s)	Z-rel (m2s)
0.641	3.420	0.286	1.114	4.103	0.204
0.663	3.158	0.356	0.822	4.237	0.225
0.688	2.862	0.429	0.602	4.152	0.203
0.717	2.538	0.471	0.429	3.566	0.105
0.748	2.198	0.412	0.290	2.359	0.004
0.785	1.853	0.271	0.181	1.155	0.101
0.827	1.513	0.110	0.098	0.329	0.436
0.878	1.190	0.027	0.042	0.039	0.818
0.938	0.895	0.008	0.013	0.008	0.964
1.013	0.634	0.005	0.005	0.009	0.793
1.110	0.417	0.002	0.003	0.002	0.464

The energy spectra of the waves and the four motions of point B based on the frequency of encounter are:

ENC FREQ (r/s)	WAVE (m2s)	X-abs (m2s)	Y-abs (m2s)	Z-abs (m2s)	Z-rel (m2s)
0.828	2.161	0.181	0.704	2.593	0.129
0.863	1.971	0.222	0.513	2.643	0.140
0.904	1.760	0.264	0.370	2.554	0.125
0.950	1.537	0.286	0.260	2.160	0.063
1.003	1.308	0.245	0.173	1.404	0.003
1.065	1.081	0.158	0.105	0.674	0.059
1.138	0.864	0.063	0.056	0.188	0.249
1.227	0.662	0.015	0.023	0.022	0.455
1.338	0.483	0.004	0.007	0.005	0.521
1.480	0.330	0.003	0.002	0.005	0.413
1.670	0.207	0.001	0.002	0.001	0.231

Exercise 13. Resistance and Drift Forces in Irregular Waves.

A sphere-shaped structure with a radius R = 1.00 meter floats exactly half-immersed (hemisphere) in a deep fresh water lake ($r_{water} = 1000$ kg/m^3) with still water.

Suddenly, a little storm rises. The wind speed just above the water surface is 1.00 m/s.
After a while, this wind results in a significant wave height $H_{1/3}$ of 0.10 meter and an average zero-crossing period T_2 of 0.75 seconds. The energy spectrum of these little waves $S_z(w)$ is presented in Figure 3.

According to the diffraction program DELFRAC of Prof. Pinkster, this floating body has a transfer function of the mean second-order horizontal drift forces as given in Figure 3 (with $g = 9.81$ m/s^2).

Caused by the combined action of wind, waves and water, the sphere reaches after some time a mean translation velocity V_{sphere}. The drag coefficient C_D of the hemisphere is 0,40 (based on the front area).
De density of air is 1/800 of the density of water ($r_{air} = r_{water} / 800$).

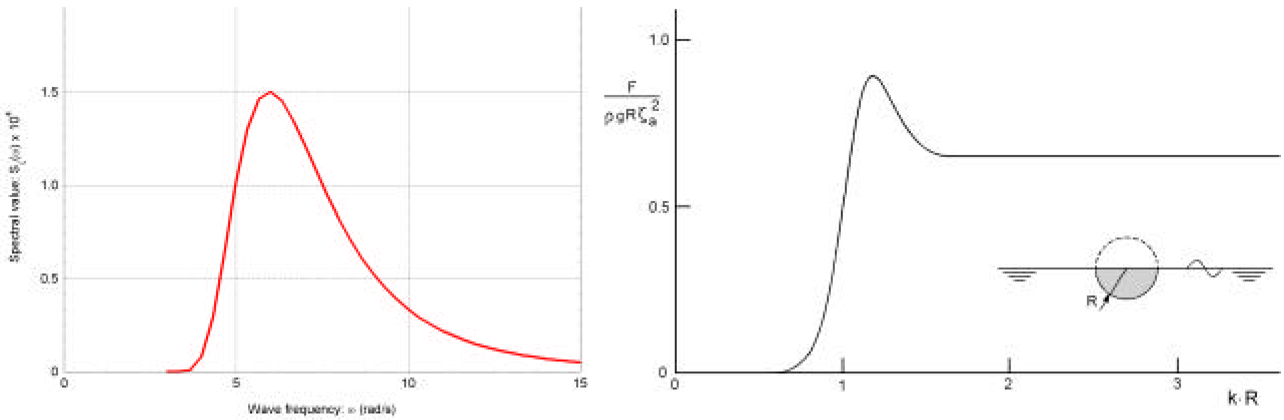

Figure 3: Wave spectrum and transfer function of the mean second-order horizontal drift forces on a hemisphere

<u>Exercise:</u>
Determine the approximated final mean velocity V_{sphere} of the sphere.

<u>Solution:</u>

$V_{sphere} \approx 0.16$ m/s.

Exercise 14. Estimation of Mean Second-Order Wave-Drift Forces.

A dynamic positioned FPSO has a length $L_{pp} = 232.00$ m, a breadth $B = 42.00$ m and a draught $d = 14.25$ m.
The vessel is subject to irregular deep-ocean beam waves, defined by $H_{1/3} = 2.00$ m and $T_2 = 7.0$ s.

Exercise:
Give a rough estimation of the mean second-order wave-drift forces on the vessel.
(Hint: Consider the hull of the tanker being a vertical wall of infinite depth.)

Solution:
$\bar{F}_{drift} \approx 583$ kN.

Exercise 15. Turning Circle Manoeuvre.

The main dimensions of a general purpose cargo vessel are:

Length	L	= 140.00 m
Breadth	B	= 18.20 m
Draught	d	= 7.82 m
Block coefficient	C_b	= 0.673
Projected rudder area	A_R	= 14.60 m^2.

The ship sails with a constant initial speed of 17.0 knots (1 knot = 0.5144 m/s) in calm deep water. All effects of currents, wind and waves may be ignored. The centre of gravity, G, is supposed to be situated at half the ship's length.

At time $t = t_0$, the ship's captain starts a turning circle manoeuvre. The rudder will be set from initially 0^0 to a constant rudder angle of 20^0, reached at $t = t_1$, with an almost constant rudder speed of 2.0 ^{0}/s.
After turning 360^0 at $t = t_2$ = 460 s, the ship has reached a steady rate of turn with $\dot{y}$ = 0.90 ^{0}/s and the forward ship speed has dropped to 11.9 knots. GPS measurements on the ship show a drift angle of 8.2^0.

Exercise:
Determine from these data:
1. The turning circle diameter, D_c.
2. The drift velocity, v.
3. The position, L_P, of the so-called neutral or pivoting point, P, forward half the ship length. This is that point, along the length of the ship, at which an applied force (ignoring transient effects) does not cause the ship to deviate from a constant heading.
4. The local (or kinematic) drift velocity near the rudder (x_R=-70 m), v_R.
5. The local (or kinematic) drift angle near the rudder, $\boldsymbol{b}_R^*$.
6. The effective rudder angle, $\boldsymbol{d}^*$.
7. The first order Nomoto coefficients, K and T.
8. The turning circle diameter for a rudder angle of 30^0, according to Nomoto.

Solution:
1. D_c = 780 m.
2. v = -0.87 m/s.
3. L_P = 56 m.
4. v_R = -1.97 m/s
5. $\boldsymbol{b}^*$ = 17.9^0.
6. $\boldsymbol{d}^*$ = 2.1^0.
7. K = 0.045 s^{-1} and T = 50 s.
8. D_c = 520 m.

Exercise 16. Directional Stability.

Two different ship designs, A and B, have each length $L =$ 200 m. Suppose that the centres of gravity, G, are situated at half the ship length.
Model experiments have provided (among others) the following manoeuvring coefficients:

	$m^{'}$	$Y_v^{'}$	$N_v^{'}$	$Y_r^{'}$	$N_r^{'}$
Design A	0.12	-0.36	-0.07	0.06	-0.07
Design B	0.10	-0.26	-0.10	0.01	-0.03

The terms in the equations of motion are made non-dimensional with the density of water $\boldsymbol{r}$, the ship length L and the forward ship speed U, forces by $1/2\, \boldsymbol{r} U^2 L^2$ and moments by $1/2\, \boldsymbol{r} U^2 L^3$.

Exercise:

1. Give the three dimensional linear equations of motion in the horizontal plane of the steered ship(s) at a straight course.
2. Give these equations in a non-dimensional form, as they will be used in stability analyses.
3. Derive the conditions that have been fulfilled, for judging the directional stability and comment on the directional stability of each of these two ship designs.
4. How far are the neutral (or pivoting) points, P, of the two ships forward half the ship's length?

Solution:

1. Equations of motion:

$$\left(m - X_{\dot{u}}\right)\cdot \dot{u} - X_u \cdot \Delta u = 0$$

$$\left(m - Y_{\dot{v}}\right)\cdot \dot{v} - Y_V \cdot v - Y_{\dot{r}} \cdot \dot{r} - \left(Y_r - m \cdot U\right)\cdot r = Y_d \cdot \boldsymbol{d}$$

$$-N_{\dot{v}} \cdot \dot{v} - N_V \cdot v + \left(I_z - N_{\dot{r}}\right)\cdot \dot{r} - N_r \cdot r = N_d \cdot \boldsymbol{d}$$

2. Equations of motion:

$$\left(m^{'} - X_{\dot{u}}^{'}\right)\cdot \dot{u}^{'} - X_u^{'} \cdot u^{'} = 0$$

$$\left(m^{'} - Y_{\dot{v}}^{'}\right)\cdot \dot{v}^{'} - Y_V^{'} \cdot v^{'} - Y_{\dot{r}}^{'} \cdot \dot{r}^{'} - \left(Y_r^{'} - m^{'}\right)\cdot r^{'} = 0$$

$$-N_{\dot{v}}^{'} . \dot{v}^{'} - N_V^{'} \cdot v^{'} + \left(I_z^{'} - N_{\dot{r}}^{'}\right)\cdot \dot{r}^{'} - N_r^{'} \cdot r^{'} = 0$$

3. All three stability roots $\boldsymbol{s}_{1,2,3}$' should be negative.

   Design A is stable: $\boldsymbol{s}_1 < 0$ and $\boldsymbol{s}_{2,3} < 0$ because $A < 0$, $B > 0$ and $C > 0$ (= +0.0210) in equation $A \cdot \boldsymbol{s}_i'^2 + B \cdot \boldsymbol{s}_i' + C = 0$.

   Design B is unstable: $\boldsymbol{s}_1 < 0$ and $\boldsymbol{s}_{2,3} > 0$ because $A < 0$, $B > 0$ and $C < 0$ (= -0.0012) in equation $A \cdot \boldsymbol{s}_i'^2 + B \cdot \boldsymbol{s}_i' + C = 0$.

4. Design A: $x_P = 38.90$ m.

   Design B: $x_P = 76.90$ m.