Questions on Ship and Offshore Hydromechanics

(in progress of formation)

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Contents:

Question 1. Standing Waves

The velocity potential of a simple regular deep-water wave is given by:

$$\Phi = \frac{\zeta_a \cdot g}{\omega} \cdot e^{-k \cdot z} \cdot \sin\left(k \cdot x - \omega \cdot t\right)$$

a) Using this definition, determine the velocity potential of a standing wave.
b) Determine for both, the simple wave and the standing wave, the paths of the fluid particles.
c) Sketch the wave elevation of the standing wave as a function of x and t.
   How large is its amplitude?
d) Determine for both, the simple wave and the standing wave, the energy in the waves per unit wave surface area.

Question 2. Regular Wave Observation

The mast of a small floating raft is observed to oscillate with a period of 7.0 seconds and amplitude from the vertical of $\pm$ 8.0 degrees, due to the passage of a train of (more or less regular) deep-water waves.
Find of these waves:
a) wave height ($H =$ 3.39 m).
b) wave length, ($\lambda =$ 76.5 m).
c) phase velocity of waves ($c =$ 10.9 m/s).

Question 3. Axes Systems in Ship Motion Calculations

a) Mention and explain the three axes systems, used in ship motion calculations.
b) Waves are defined in an earth-bound axes system, $S(x_0, y_0, z_0)$. Determine the wave elevation in the steadily translating axes system, $O(x, y, z)$.
c) Determine the general relation between the frequency of encounter, ω_e, and the wave frequency, ω.

Question 4. Load Superposition

The loads on a body - oscillating in waves - are divided in two separate contributions.
a) Mention and explain these two contributions.
b) Show these separate contributions in an uncoupled equation of motion for heave.
c) Explain the terms "hydrodynamic mass" and "hydrodynamic damping".

Question 5. Dissipation of Energy

The hydromechanical loads on a floating structure consist of inertia, damping and spring terms.
a) Show for translations, such as heave motions, that only the damping term dissipates energy.
b) Show the same for rotations, such as roll motions.
c) Show, in case of a non-linear damping behaviour, how an equivalent linear damping coefficient can be obtained.

Question 6. Model Tests

a) Describe briefly model experiments to determine the hydrodynamic mass and damping of a heaving vertical cylinder by a free decay test.
Explain - by presenting the relevant equations - how these coefficients can be determined from the measured motion signal.

b) Describe briefly model experiments to determine the hydrodynamic mass and damping of a heaving vertical cylinder by forced oscillation tests.
Explain - by presenting the relevant equations - how these coefficients can be determined from the measured force signal.

c) Describe briefly model experiments to determine the wave load amplitude and phase lag of a heaving vertical cylinder.
Explain - by presenting the relevant equations - how these phenomena can be determined from the measured wave load signal.
Explain and determine the magnitude of the so-called Froude-Krylov contribution in these loads.

Question 7. Equation of Motion

Consider a vertical cylinder - diameter D and draft T, upright in water with density ρ - heaving in regular waves with amplitude ζ_a and frequency ω. The hydrodynamic mass, $a(\omega)$, and damping, $b(\omega)$, are known.

a) Give the equation of motion for heave.
b) The spring coefficient, c, follows from the dimensions given above. Show this.
c) Determine at frequency ω the wave load amplitude, F_a / ζ_a, and phase shift, ε_{Fz}.
d) Determine at frequency ω the heave amplitude, z_a / ζ_a, and phase shift, ε_{zz}.
e) Determine at frequency ω the relation between the heave motion and the wave load, z_a / F_a and ε_{zF}.
f) Determine the natural frequency for heave, ω_0.

Question 8. Heave Resonance of a Vertical Cylinder

Consider a circular cylinder with a radius R, floating upright with a draft T in water with a density ρ.

a) Give an approximation of the wave length in short (deep water) waves at which resonance in heave can occur.
b) Discuss the effect of damping on the frequency characteristics of heave at different R/T ratios.

Question 9. Irregular Motions

a) Show how one can determine the heave spectrum, $S_{zz}(\omega)$, when the transfer function of the motion, $z_a / \zeta_a(\omega)$, and the wave spectrum, $S_\zeta(\omega)$, are given. Prove the relation used for this.

b) Define the Root Mean Square value, $z_{a\,RMS}$, of this motion, the significant heave amplitude, $z_{a\,1/3}$, the mean period based on the centroid of the spectrum, T_{1z}, and the mean zero-crossing period, T_{2z}.
c) Determine the probability that the significant heave amplitude, $z_{a\,1/3}$, will be exceeded in this sea state defined by $S_z(\omega)$.
d) Show how this heave spectrum based on a circular frequency ω can be transferred to a spectrum based on the frequency f in Hz. Show these spectra in one realistic graph.
e) Give the relation between the heave velocity spectrum, $S_{\dot{z}z}(\omega)$, and the heave displacement spectrum, $S_{zz}(\omega)$.
f) Give the relation between the heave acceleration spectrum, $S_{\ddot{z}z}(\omega)$, and the heave displacement spectrum, $S_{zz}(\omega)$.
g) Which of the zero, first and second order moments m_0, m_1 and m_2 of the displacement, velocity and acceleration spectra are similar?

Question 10. Irregular Motions of a Spar Buoy

The response amplitude operator of the heave motion, $z_a / \zeta_a(\omega)$, of a Spar buoy is known from 3-D calculations. The wave spectrum of the irregular waves, $S_\zeta(\omega)$, is given too.

Using this, show how one can determine:

a) the significant amplitude and average zero-upcrossing period of the:
   - vertical relative displacements
   - vertical absolute displacements
   - vertical absolute velocities
   - vertical absolute accelerations
b) the probability of exceeding a threshold value a by the vertical accelerations
c) the number of times per hour that this will happen.

Question 11. Mean Wave Loads on a Wall

The velocity potential and elevation of a regular wave in deep water - progressing in the positive x direction - are given by:

$$\Phi = \frac{\zeta_a \cdot g}{\omega} \cdot e^{-k \cdot z} \cdot \sin(k \cdot x - \omega \cdot t)$$

$$\zeta = \zeta_a \cdot \cos(k \cdot x - \omega \cdot t)$$

The time-averaged wave load on a vertical wall - in regular waves perpendicular to this wall - is:

$$\frac{1}{2} \cdot \rho \cdot g \cdot \zeta_a^2 \qquad \text{per meter length of the wall}$$

a) Show the principle of the determination of this value.
b) How large is this load in irregular waves with a significant wave height $H_{1/3}$?
c) What assumption has been made when this formula is applied as an approximation for the mean wave loads on a crude oil carrier, moored in beam waves?
   Is the actual load on the ship lower or higher than this approximation? Why?

Question 12. Wave Drift Forces

Floating bodies in waves are not only loaded by oscillating forces with frequencies in the wave frequency range, but also by mean second order forces as well as low-frequency forces in irregular waves.

a) Which simple experiment proofs the presence of a mean wave drift force on a body in regular waves?
   Give an explanation for the presence of this force.

According to Maruo and to others, the mean (time-averaged) wave drift force in regular beam waves per unit length can be defined by:

$$F'_d = \frac{1}{2} \cdot \rho \cdot g \cdot \left\{ R(\omega) \cdot \zeta_a \right\}^2$$

in which $R(\omega)$ is a reflection coefficient and ζ_a is the amplitude of the incident regular wave.

b) Show, using the potential theory, the validity of Maruo's expression with $R(\omega) = 1.0$ for the case of a vertical wall.

Consider now the case of a moored tanker in irregular head waves.

c) Explain the low-frequency behaviour in irregular waves of the wave drift forces on this tanker.
d) Explain why - nevertheless that second order wave drift forces are small when compared to first order wave forces - relatively high second order motions can appear in irregular waves.
e) Sketch a time history of the surge motion in irregular waves and show in this figure the various causes, which lead to this motion.

Question 13. Cummins Equations

The so-called Cummins-equation in the time domain is given by:

$$(M + A) \cdot \ddot{x}(t) + \int_0^\infty B(\tau) \cdot \dot{x}(t - \tau) \cdot d\tau + C \cdot x(t) = X(t)$$

with:

$$B(\tau) = \frac{2}{\pi} \cdot \int_0^\infty b(\omega) \cdot \cos(\omega\tau) \cdot d\omega$$

$$A = a(\omega) + \frac{1}{\omega} \cdot \int_0^\infty B(\tau) \cdot \sin(\omega\tau) \cdot d\tau$$

in which $a(\omega)$ and $b(\omega)$ are the hydrodynamic mass and damping determined in the frequency domain.

a) Which term(s) in this Cummins-equation are linear en which one(s) can be non-linear?
b) Discuss the physical meaning of the individual terms.
c) Because many computer programs fail at very high frequencies, the high-frequency "tail" of the damping curve, $b(\omega)$, is often approximated by $= \beta / \omega^3$, where β is determined from the damping at the highest successful frequency in the calculations.
d) Explain the origin of this approximation and reproduce a derivation of the used relation.
e) Show, given the frequency characteristics of the wave force, $F_a / \zeta_a(\omega)$, and a wave spectrum, $S_\zeta(\omega)$, how a time history of the (linear) wave force can be generated.
f) How can the effect of a non-linear spring be added to this Cummins-equation?
g) How can this Cummins-equation be solved?
h) Discuss the mutual "pros and cons" of frequency-domain and time-domain equations.

i) Give three examples of a problem that can not be solved in the frequency-domain, by which one is forced to solve this problem in the time-domain

Question 14. Potential Theory Applications

Consider the total velocity potential to describe the motions of the fluid particles around an oscillating body in waves.

a) In which three individual parts can this potential be divided and why is this division permitted? Why is the hydrostatic contribution ignored?
What is for each of these potentials the motive for the appearance of the related motions of the water particles.
Discuss the division in space and time dependent terms.

b) Discuss and explain the requirements and boundary conditions, that have to be fulfilled when defining these velocity potential(s).
Which of these requirements or boundary conditions are similar to those used when defining the velocity potential of a simple regular wave?

c) What is the relation between the radiation potential and the hydrodynamic mass and damping coefficients?
Which symmetry properties hold for the hydrodynamic mass and damping coefficients? From which appears this?

d) Explain why the integration of the radiation potentials over a large closed surface of a volume with fluid can be simplified to integration over the wetted surface of the body only.

e) Give the expression for the wave loads based on an integration of the pressures on the body according to the linearized Bernoulli equation.

f) Explain why $+\partial\Phi_w / \partial n$ is equal to $-\partial\Phi_d / \partial n$ in the definition of the wave loads.

g) The so-called "Haskind relations" provide the opportunity to calculate the wave loads without using the diffraction potential. Whereupon is this based?

h) Explain the physical meaning of the potential mass coefficients M_{35} and M_{53} and their related hydrodynamic force and moment.

i) Explain why the potential mass coefficients M_{23} and M_{32} and their related hydrodynamic forces do not exist for a crude oil carrier.

Question 15. 2-D Velocity Potentials of Ursell and Tasai

Consider a cross section of a horizontal cylinder, oscillating in the surface of a fluid.

a) Ursell assumed two types of waves, which were produced by this oscillating cylinder.
Describe the physical characteristics of these two wave systems.

b) Which potential flow element combinations (source, sink, doublet, etc.) are used for the simulation of the vertical motions of this cylinder and which ones for the simulation of the horizontal motions.
Explain the different approaches for the vertical and horizontal motions briefly.

c) Which requirements and boundary conditions are fulfilled already by these potentials and which one(s) are depending on the particular cross section of the body itself.

d) Describe how the potential mass and damping for heave can be determined, as the velocity potential is known.

e) Describe some cases where the conformal mapping method will fail?

f) Describe the (non-) similarities of a "Lewis" cross section and the actual cross section.

g) Determine the Lewis coefficients (M_s, a_1 and a_3) of half a circular cross section with a diameter of 10 m.

h) Describe the principle of a method, which can be used to obtain the N-parameter close-fit conformal mapping coefficients of a cross section?

i) Consider now a rectangular cross section.
Explain why a Lewis transformation gives much more poor results for roll than it gives for sway and heave.

Question 16. 2-D Velocity Potentials of Frank

a) Based on work of Wehausen and Laitone, the complex potential at z of a pulsating point-source of unit strength at the point ζ in the lower half plane of a cross section was given by Frank as:

$$G^*(z,\zeta,t) = \frac{1}{2\pi} \cdot \left\{ \ln(z-\zeta) - \ln(z-\overline{\zeta}) + 2 \cdot PV \int_0^\infty \frac{e^{-ik\cdot(z-\overline{\zeta})}}{\nu - k} \cdot dk \right\} \cdot \cos(\omega \cdot t) - \left\{ e^{-ik\cdot(z-\overline{\zeta})} \right\} \cdot \sin(\omega \cdot t)$$

where:

$$z = x + iy \qquad \zeta = \xi + i\eta \qquad \overline{\zeta} = \xi - i\eta$$

In here:

ζ is defined in the lower half plane of the cross section

$\overline{\zeta}$ is defined in the upper half plane (mirrored)

$\nu = \omega^2 / g$ is the wave number in deep water

$PV \int_0^\infty \frac{e^{-ik\cdot(z-\overline{\zeta})}}{\nu - k} \cdot dk$ is a principle value integral with a variable k.

Discuss the possible singularities in this Green's function.

b) In which cases would one prefer Frank's pulsating source method above the method of Ursell and Tasai with 10-parameter conformal mapping?

c) What is the major disadvantage of Frank's method, when compared with the method of Ursell and Tasai?

Question 17. 2-D versus 3-D Approaches

a) Mention some practical examples in offshore applications for which a 2-D approach suffices and some examples for which a 3-D approach is required.

b) Discuss the mutual (dis)advantages of the 2-D and the 3-D approaches when determining the potential coefficients.

Question 18. Forced Oscillation Tests

a) Describe briefly model experiments to verify the results of potential calculations on the hydrodynamic mass and damping for a heaving and pitching crude oil carrier for a range of frequencies.

b) What are the scale-factors for these coefficients.

c) Explain how - by using a Fourier analysis - the in-phase and out-of-phase terms can be found from the measured signal.

Question 19. Wave Load Measurements

Describe briefly model experiments to verify the results of wave load calculations for a heaving and pitching crude oil carrier.

Question 20. Symmetric and Anti-Symmetric Motions

Sometimes a distinction will be made between symmetric and anti-symmetric motions of ships.

a) Explain the difference between these two types of motion.
b) What is the effect of this distinction on the equations of motion of a ship for 6 degrees of freedom?
c) What can a possible influence of mooring be on this distinction?

Question 21. Experimental Determination of Mass and Damping

The frequency dependent hydrodynamic mass and damping for heave and pitch of a floating structure can be determined experimentally by means of model tests.

a) Describe to types of model experiments for this purpose.
   What are the mutual "pros and cons" of these two methods?
b) Which scaling law(s) have to be used.
c) The model is scaled by α.
   Determine the scale factors of the time, the amplitude and frequency of oscillation, the forward speed, the length of the radiated waves, the measured forces, moments and phase shifts and the obtained hydromechanical coefficients.
d) When plays the so-called "scale-effect" a significant role?

Question 22. Viscous Roll Damping

a) Explain why (generally) viscous damping of normal ships plays a role in the roll motions and not in the heave or pitch motions.
   In which case(s) can the viscous part of the roll damping be ignored?
b) Describe the various causes of viscous roll damping.
c) Sketch and explain the relation between the roll damping and the breadth to draft ratio, B/T, of a ship.

Question 23. Froude-Krilov Force for Heave

a) Discuss the so-called Froude-Krilov force for heave on a cross section of a ship.
b) Where on the cross section is the center of this force located?

Question 24. Coupled Heave and Pitch Motions

Consider the coupled equations of motion for heave and pitch of a floating structure in regular head waves.

a) Reproduce these two equations of motion. When are the so-called coupling terms in these equations zero and explain why.

b) In which case(s) has the use of the strip theory a great advantage on the use of the 3-D diffraction method and in which case(s) has the last one preference?
c) Which methods can be used in the strip theory to calculate the hydromechanical coefficients of a cross section? Discuss the benefits and limitations of each method.
d) Describe briefly model experiments to verify the results of calculations on the hydrodynamic mass and damping for a heaving and pitching crude oil carrier.
e) Reproduce the calculation principle of the three terms in the wave loads on a ship, as used in the strip theory method. Hint: mind the relative velocity principle.
f) Describe briefly model experiments to verify the results of wave load calculations for a heaving and pitching crude oil carrier

Question 25. Natural and Resonance Frequencies

a) A box-shaped vessel has a length of 50.00 m, a beam of 12.00 m, a draft of 3.00 m and a freeboard 6.00 m. The height of the CoG above the bottom, $\overline{KG}$, is 4.50 m. Assuming that the vessel's mass is uniformly distributed throughout the length and section of the vessel, and neglecting the effects of associated water, calculate the natural periods of heave, roll and pitch in salt water ($\rho = 1.025$ t/m^2.
b) A vessel, length 200 meter, whose natural periods of roll and pitch are 12.5 and 10.0 seconds respectively, is sailing at 20 knots in a (more or less regular) sea with a wave length of 250 meter. Calculate the headings on which the largest roll and pitch amplitudes are likely.
c) When the amplitude characteristics of for instance heave shows resonance at a certain frequency of oscillation, this frequency is not necessarily the natural frequency of the floating structure. Explain this statement.

Question 26. Free-Surface Anti-Rolling Tank

The uncoupled equation of motion for roll of an FPSO in regular beam waves is given by:

$$\left(I_{xx}+a_{44}\right)\cdot\ddot{\phi}+b_{44}\cdot\dot{\phi}+c_{44}\cdot\phi = X_{4a}\cdot\cos\left(\omega\cdot t+\varepsilon_{X_{4a}\zeta}\right)$$

while the waves are given by:

$$\zeta = \zeta_a\cdot\cos\left(\omega\cdot t\right)$$

Because of too heavy roll motions, it is decided to investigate the reduction of roll by a free-surface anti-rolling tank in the vessel.
The harmonic exciting moments on its environment of this anti-rolling tank is given by:

$$K = K_a\cdot\cos\left(\omega\cdot t+\varepsilon_{K\phi}\right)$$

The amplitude K_a/ϕ_a and phase lag $\varepsilon_{K\phi}$ as function of ω has been determined by forced roll oscillation tests with a tank model.
Show how the effect of this tank can be included in the coefficients, in the left hand side of the equation of motion for roll of the FPSO.

Question 27. Added Resistance

The resistance increase of a ship sailing in regular waves can be obtained by model experiments or be computed. Then, with a known wave energy spectrum, the mean added resistance in irregular waves can be found.

a) What is the relation between the mean added resistance in regular waves, $R_{aw}(\omega_e)$, and the wave amplitude, ζ_a?
Is this a linear or a non-linear relation? How can this be concluded from results of model tests?

b) Show the principle of the calculation of the mean added resistance in regular waves either by:
- the radiated energy method (Gerritsma \& Beukelman), and
- the integrated pressure method (Boese).

c) Determine the relation between the mean added resistance in regular waves - transfer function $R_{aw}/\zeta_a^{\,2}(\omega_e)$ - and the mean added resistance in irregular waves, R_{AW}, as the wave energy spectrum, $S_\zeta(\omega)$, is given.

d) Determine the relation between the mean added resistance in irregular waves, R_{AW}, and the significant wave height, $H_{11/3}$ of a Bretschneider wave spectrum.

Question 28. 3-D Diffraction Computations

3-D diffraction calculations are carried out to determine the behaviour of arbitrarily shaped floating structures in waves; the computation method is based on the classic potential theory.

a) Which velocity potentials play a role here and describe the physical background of each of these potentials.

b) How are the potentials described which are related to the presence of the floating body?
Give a general equation of the potential as a distribution of sources over the mean wetted surface of the body and explain the different terms in this equation.
Which magnitudes in here can be calculated directly and which one(s) are unknown so far?

c) The formulation of the Green's function (or influence function) in this equation is chosen in such a way that these potentials fulfil automatically a number of requirements or boundary conditions.
Mention these requirements or conditions.

d) Based on which - still missing - boundary condition are the source strengths being calculated and which (integral-)equation needs then to be solved?
How can this equation be discretized - to restrict the number of unknowns - and what does this mean for the description of the (under water) hull form of the body?

e) As the solutions of the source strengths have been found, then a number of magnitudes - required for determining the body motions - can be calculated. Mention these magnitudes.

f) Give the linearized equations of motion of the floating body and explain the physical meaning of the different terms in these equations.

Question 29. Non-Linear Behaviour of Ships

For the determination of the first order wave loads, the "body-in-waves" has no motions (zero order) in the earth-bound axis system. The waves are approaching a restrained body.
For the determination of the second order wave loads, the "body-in-waves" is carrying out a first order harmonic motion, forced by first order wave loads.
The waves are approaching a harmonic oscillating body and - using perturbation methods - the orientation of a surface element relative to the fixed $O(X_1, X_2, X_3)$ system of axes becomes:

$$\vec{N} = n + \varepsilon \vec{N}^{(1)}$$

The parameter ε is some small number, with $\varepsilon << 1$, which denotes the order of oscillation.
If the velocity potential:

$$\Phi = \varepsilon\Phi^{(1)} + \varepsilon^2\Phi^{(2)} + ...$$

is known, the fluid pressure at a point is determined using the Bernoulli equation:

$$p = -\rho \cdot g \cdot X_3 - \rho \cdot \frac{\partial \Phi}{\partial t} - \frac{1}{2} \cdot \rho \cdot \left(\vec{\nabla} \Phi \right)^2$$

Assuming that this point is carrying out small - first order - wave frequency motions, $\vec{X}^{(1)}$, about a mean position, $\vec{X}^{(0)}$, and applying a Taylor expansion to the pressure in its mean position, yields:

$$p = p^{(0)} + \varepsilon p^{(1)} + \varepsilon^2 p^{(2)}$$

The instantaneous wetted surface, S, is split into two parts: a constant part, S_0, up to the static hull waterline and an oscillating part, s, the splash zone between the static hull waterline and the wave profile along the body.

a) Show (without presenting very detailed formulas) that the fluid force on the oscillating body can be split into three parts: a hydrostatic fluid force, $\vec{F}^{(0)}$, a first order oscillatory fluid force, $\vec{F}^{(1)}$ and a second order fluid force, $\vec{F}^{(2)}$.
b) Discuss this approach and its results.

Question 30. Mean Wave Drift Forces

Floating structures in waves are not only loaded by oscillating forces with frequencies in the wave frequency range, but also by mean second order forces as well as low-frequency forces in irregular waves.

a) What does these different loads mean for:
   - the resulting ship motions (show the motion components), and
   - the mooring forces.
b) What is the relation between the mean drift force in regular waves and the wave amplitude? Is this a linear or a non-linear relation?
   How can this be concluded from results of model tests?
c) Determine the relation between the mean drift forces in regular waves (transfer function) and the mean drift forces in irregular waves as the wave energy spectrum, $S_\zeta(\omega)$, is given.
d) Determine the relation between the mean drift force in irregular waves and the significant wave height.
e) Drift forces can be calculated by using the potential theory.
   - Show the principle of this calculation.
   - Which terms in the mean drift forces are similar to the added resistance terms calculated by the integrated pressure method (Boese).

Question 31. Wave Set Down

Discuss (by using a practical example) the phenomenon "wave set-down" and its effect on the heave motions of a vessel.

Question 32. Stiffness of a Mooring System

Discuss the phenomenon that as the stiffness of a mooring system increases, so does the RMS value of the low frequency mooring force too.

Question 33. Station Keeping

Thrusters may be used to keep a vessel in its desired position.

a) How can the propulsive characteristics of a thruster be presented?
b) An important thruster-hull interaction effect is the Coanda effect.
Explain this Coanda effect and its influence on the efficiency of the thruster.
c) Show how the thruster forces of a dynamic position system are accounted for in the equation of motion for surge of a floating structure in waves.
d) Discuss the problems that can appear when a thruster of a dynamic positioned floating structure in waves comes too close to the free surface.
e) For tunnel thrusters, the thruster forces are affected by the flow of a current past the inlet and outlet. Discuss the effect of this current on the performance of a bow thruster.

Question 34. Sustained Sea Speed

A crude oil carrier is sailing in a rough to very rough seaway.

a) Explain briefly - for a crude oil carrier sailing in irregular waves - the different resistance and propulsion components, which cause:
   - involuntary speed reduction
   - voluntary speed reduction.
b) Discuss the phenomena (related to the ships motions) which can be reason for the ship's captain to reduce speed and/or change heading. Account in your considerations for the loading of the ship, fully loaded or ballast condition.