3. Float-On Float-Off Pontoon

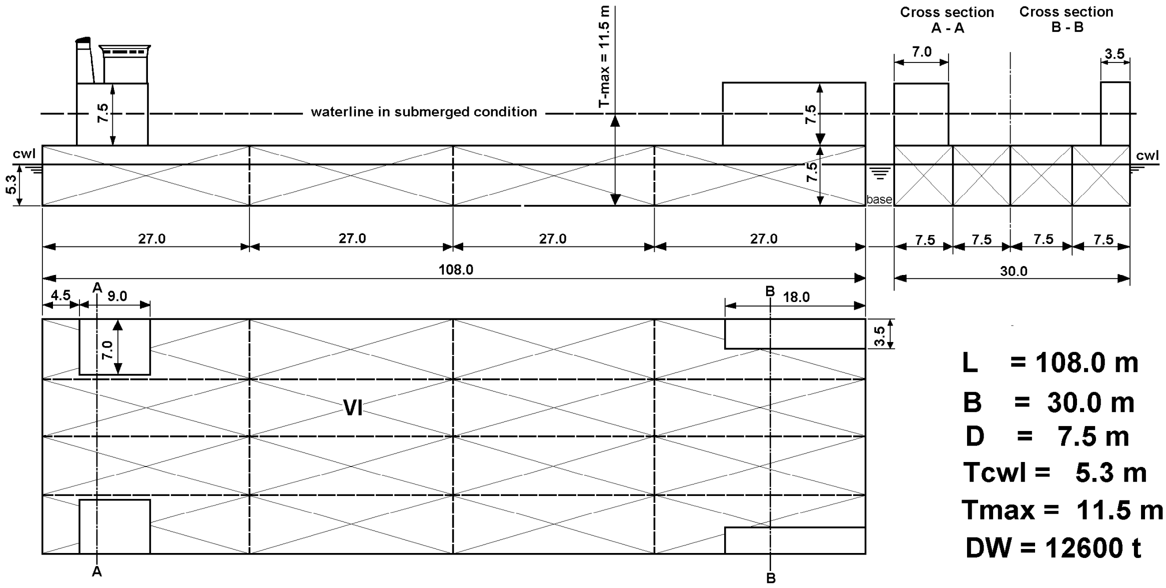

Figure 0.1: Float-On Float-Off Pontoon

In a harbor with fresh water ($\rho = 1.000$ ton/m^3) an empty rectangular Float-On Float-Off Pontoon, as given in figure 0.1 has an amidships draft of 1.55 meter in an upright even keel condition. A rough inclining experiment has been carried out, by fully filling a port side tank aft (tank VI), which is bounded by half the length of the pontoon and the longitudinal middle line plane of the pontoon, with fresh water ($\rho_{wb} = 1.000$ ton/m^3). The measured angle of heel ϕ_1 was 1.46 degrees. In the calculations, the volumes of plating, frames and other structure parts may be ignored.

a) Determine the position of the centre of gravity G_0 and the initial metacentric height $\overline{G_0M_0}$ of the empty pontoon.

b) Determine the trim angle θ_1 during the inclination experiment.

c) Determine the drafts at the four corners of the pontoon during the inclination experiment.

d) Determine the angle of heel ϕ_2 in case of an 80 per cent filled tank, during the inclination experiment.

During operation, the pontoon will be sunk down in a protected bay by loading ballast seawater ($\rho = \rho_{wb} = 1.025$ ton/m^3) in all 16 tanks. This ballast water is supposed to have an equal height h in all 16 tanks.

e) Determine the initial metacentric height $\overline{GM}$ at an even keel draft of 7.50 meter, supposing that water will just not cover the deck.

f) Determine the $\overline{GM}$-value at an even keel draft of 7.50 meter, supposing that water had just covered the deck.

g) Determine the $\overline{GM}$-value, when the pontoon has been sunk down until an even keel draft of 11.50 meter.

Now, the pontoon picks up a drill-rig, with the following specifications:
– upright even keel condition
– mass = 4920 ton
– $\overline{KG}$ = 20.00 meter
– water plane dimensions: 40 x 40 meter
– no free surfaces of liquids in any tank

h) Determine the initial metacentric height of the rig.

i) Determine the initial metacentric height of pontoon+rig, supposing that they just hit each other when de-ballasting the pontoon during loading of the rig (centre of rig above centre of pontoon).

j) Determine the initial metacentric height of pontoon+rig when all water ballast has been removed from the pontoon.

k) Determine the angle of heel when, due to an inaccurate loading of the pontoon, the centre of the rig is amidships but 1.0 meter outside the middle line plane of the pontoon.

Solutions:

a) G_0: amidships at middle line plane with $\overline{KG_0} = 4.02$ m and $\overline{G_0M_0} = 45.15$ m.

b) $\theta_1 = 0.38^0$

c) T_{1i} = 1.28, 2.00, 2.04 and 2.76 m, respectively.

d) $\phi_2 = 1.16^0$.

e) $\overline{GM} = 9.93$ m.

f) $\overline{GM} = 1.55$ m.

g) $\overline{GM} = 1.62$ m.

h) $\overline{GM} = 25.94$ m.

i) $\overline{GM} = 5.51$ m.

j) $\overline{GM} = 10.86$ m.

k) $\phi = 2.61^0$.

Detailed Solutions

Volume of displacement ∇_0 of empty pontoon:

$$\begin{aligned} \nabla_0 &= L \cdot B \cdot T_0 \\ &= 108.00 \cdot 30.00 \cdot 1.55 \;=\; 5022 \text{ m}^3 \end{aligned}$$

Mass of displacement Δ_0 of empty pontoon:

$$\begin{aligned} \Delta_0 &= \rho \cdot \nabla_0 \\ &= 1.000 \cdot 5022 \;=\; 5022 \text{ ton} \end{aligned}$$

Solution of Part 3-a

Determine the position of the centre of gravity G_0 and the initial metacentric height $\overline{G_0M_0}$ of the empty pontoon. The empty pontoon lies at an even keel condition. So, the centre of buoyancy of the pontoon and also the centre of gravity of the pontoon are situated amidships at half the length of the pontoon.

Mass of water ballast Δ_{wb} in a fully filled tank:

$$\begin{aligned} \Delta_{wb} &= \rho_{wb} \cdot l \cdot b \cdot h \\ &= 1.000 \cdot 27.00 \cdot 7.50 \cdot 7.50 \;=\; 1519 \text{ ton} \end{aligned}$$

Centre of gravity of water ballast in a fully filled tank with respect to half the length of the pontoon, the middle line plane and the base plane:

$$\begin{aligned} x_{wb} &= -13.50 \text{ m} \\ y_{wb} &= +3.75 \text{ m} \\ z_{wb} &= +3.75 \text{ m} \end{aligned}$$

The calculation will be carried out in two steps:

1. Suppose the water ballast concentrated on a vertical line through the centre of the water plane in a horizontal plane through the centre of gravity of the water ballast (parallel sinkage).

2. Shift the centre of gravity of the water ballast to the right position by adding a heeling moment (heel).

Step 1: Suppose a parallel sinkage.

The new draft T_1 becomes:

$$\begin{aligned} \Delta_1 &= \rho \cdot L \cdot B \cdot T_1 \;=\; 1.000 \cdot 108.00 \cdot 30.00 \cdot T_1 \;=\; 3240 \cdot T_1 \\ &= \Delta_0 + \Delta_{wb} \;=\; 5022 + 1519 \;=\; 6541 \\ \text{So} &: \quad T_1 = \frac{6541}{3240} \;=\; 2.02 \text{ m} \end{aligned}$$

Centre of buoyancy above keel $\overline{KB_1}$:

$$\overline{KB_1} = \frac{T_1}{2} = \frac{2.02}{2} = 1.01 \text{ m}$$

Metacenter above centre of buoyancy $\overline{B_1M_1}$:

$$\begin{aligned}\overline{B_1M_1} &= \frac{I_T}{\nabla_1} = \frac{\frac{1}{12}LB^3}{LBT_1} = \frac{B^2}{12 \cdot T_1} \\ &= \frac{30.00^2}{12 \cdot 2.02} = 37.13 \text{ m}\end{aligned}$$

Step 2: Add a heeling moment.
A transverse shift of the centre of gravity of the water ballast over a distance y_{wb} will result in a angle of heel $\phi_1 = 1.46^0$.
Replace this shift by a heeling moment M_H:

$$M_H = \Delta_{wb} \cdot g \cdot y_{wb} \cdot \cos\phi_1$$

The righting stability moment of the pontoon M_S is:

$$M_S = (\Delta_0 + \Delta_{wb}) \cdot g \cdot \overline{G_1M_1} \cdot \sin\phi_1$$

Because of the equilibrium $M_H = M_S$, it follows for the initial metacentric height $\overline{G_1M_1}$:

$$\begin{aligned}\overline{G_1M_1} &= \frac{\Delta_{wb} \cdot y_{wb}}{(\Delta_0 + \Delta_{wb}) \cdot \tan\phi_1} \\ &= \frac{1519 \cdot 3.75}{(5022 + 1519) \cdot \tan(1.46^0)} = 34.18 \text{ m}\end{aligned}$$

This formula for $\overline{G_1M_1}$ can be used, because ϕ_1 is very small:

$$\begin{aligned}\overline{M_1N_\phi} &= \overline{B_1M_1} \cdot \frac{1}{2} \cdot \tan^2\phi_1 \\ &= \overline{B_1M_1} \cdot 0.000325 \approx 0.00\end{aligned}$$

Herewith is the position of the centre of gravity of the pontoon including water ballast $\overline{KG_1}$ known:

$$\begin{aligned}\overline{KG_1} &= \overline{KB_1} + \overline{B_1M_1} - \overline{G_1M_1} \\ &= 1.01 + 37.13 - 34.18 = 3.96 \text{ m}\end{aligned}$$

For the empty pontoon, the position of the centre of gravity $\overline{KG_0}$ follows from the first moment of masses of the pontoon including water ballast with respect to the base plane:

$$\underset{(pontoon + wb)}{\Delta_1 \cdot \overline{KG_1}} = \underset{(pontoon)}{\Delta_0 \cdot \overline{KG_0}} + \underset{(wb)}{\Delta_{wb} \cdot z_{wb}}$$

So:

$$\begin{aligned}\overline{KG_0} &= \frac{\Delta_1 \cdot \overline{KG_1} - \Delta_{wb} \cdot z_{wb}}{\Delta_0} \\ &= \frac{(5022 + 1519) \cdot 3.96 - 1519 \cdot 3.75}{5022} = 4.02 \text{ m}\end{aligned}$$

Centre of buoyancy above keel $\overline{KB_0}$:

$$\overline{KB_0} = \frac{T_0}{2} = \frac{1.55}{2} = 0.78 \text{ m}$$

Metacenter above centre of buoyancy $\overline{B_0M_0}$:

$$\begin{aligned}\overline{B_0M_0} &= \frac{I_T}{\nabla_0} = \frac{\frac{1}{12}LB^3}{LBT_0} = \frac{B^2}{12 \cdot T_0} \\ &= \frac{30.00^2}{12 \cdot 1.55} = 48.39 \text{ m}\end{aligned}$$

Herewith is the initial metacentric height of the empty pontoon $\overline{G_0M_0}$ known:

$$\begin{aligned}\overline{G_0M_0} &= \overline{KB_0} + \overline{B_0M_0} - \overline{KG_0} \\ &= 0.78 + 48.39 - 4.02 = 45.15 \text{ m}\end{aligned}$$

Solution of Part 3-b
Determine the trim angle θ_1 during the inclination experiment.
Centre of buoyancy above keel $\overline{KB_1}$:

$$\overline{KB_1} = 1.01 \text{ m}$$

Longitudinal metacenter above centre of buoyancy $\overline{BM_L}$:

$$\begin{aligned}\overline{B_1M_{1L}} &= \frac{I_L}{\nabla_1} = \frac{\frac{1}{12}BL^3}{LBT_1} = \frac{L^2}{12 \cdot T_1} \\ &= \frac{108.00^2}{12 \cdot 2.02} = 481.19 \text{ m}\end{aligned}$$

Centre of gravity above keel $\overline{KG_1}$:

$$\overline{KG_1} = 3.96 \text{ m}$$

Herewith is the initial longitudinal metacentric height of the pontoon including water ballast $\overline{G_1M_{1L}}$ known:

$$\begin{aligned}\overline{G_1M_{1L}} &= \overline{KB_1} + \overline{B_1M_{1L}} - \overline{KG_1} \\ &= 1.01 + 481.19 - 3.96 = 478.24 \text{ m}\end{aligned}$$

A longitudinal shift of the centre of gravity of the water ballast over a distance x_{wb} will result in a trim angle θ_1.
Replace this shift by a trimming moment M_{HL}:

$$M_{HL} = \Delta_{wb} \cdot g \cdot x_{wb} \cdot \cos\theta_1$$

The longitudinal righting stability moment of the pontoon M_{SL} is:

$$M_{SL} = \Delta_0 + \Delta_{wb} \cdot g \cdot \overline{G_1M_{1L}} \cdot \sin\phi_1$$

Because of the equilibrium $M_{HL} = M_{SL}$, it follows for the trim angle θ_1:

$$\begin{aligned}\tan\theta_1 &= \frac{\Delta_{wb} \cdot x_{wb}}{(\Delta_0 + \Delta_{wb}) \cdot \overline{G_1M_{1L}}} \\ &= \frac{1519 \cdot 13.50}{(5022 + 1519) \cdot 478.24} = 0.38^0\end{aligned}$$

Solution of Part 3-c
Determine the drafts at the four angular points of the pontoon during the inclination experiment.
Half the heel displacement is:

$$\frac{B}{2} \cdot \tan\phi_1 \;=\; \frac{30.00}{2} \cdot \tan(1.46^0) \;=\; 0.38 \text{ m}$$

Half the trim displacement is:

$$\frac{L}{2} \cdot \tan\theta_1 \;=\; \frac{108.00}{2} \cdot \tan(0.38^0) \;=\; 0.36 \text{ m}$$

Drafts at angular points of pontoon:

Starboard aft:	$2.02 + 0.38 + 0.36$	$= 2.76$ m
Starboard forward:	$2.02 + 0.38 - 0.36$	$= 2.04$ m
Port side aft:	$2.02 - 0.38 + 0.36$	$= 2.00$ m
Port side forward:	$2.02 - 0.38 - 0.36$	$= 1.28$ m

Solution of Part 3-d
Determine the angle of heel ϕ_2 in case of an 80 per cent filled tank, during the inclination experiment.
Mass of water ballast Δ_{wb} in the 80 per cent filled tank:

$$\Delta_{wb} \;=\; 0.80 \cdot 1519 \;=\; 1215 \text{ ton}$$

Centre of gravity of water ballast in the 80 per cent filled tank with respect to half the length of the pontoon, the middle line plane and the base plane:

$$\begin{aligned} x_{wb} &= -13.50 \text{ m} \\ y_{wb} &= +3.75 \text{ m} \\ z_{wb} &= 0.80 \cdot 3.75 \;=\; +3.00 \text{ m} \end{aligned}$$

Suppose a parallel sinkage.
The new draft T_2 becomes:

$$\begin{aligned} \Delta_0 + \Delta_{wb} \;=\; \Delta_2 &= \rho \cdot L \cdot B \cdot T_2 \\ &= 1.000 \cdot 108.00 \cdot 30.00 \cdot T_2 \;=\; 5022 + 1215 \\ \text{So} \quad &: \quad T_2 \;=\; 1.92 \text{ m} \end{aligned}$$

Centre of buoyancy above keel $\overline{KB_2}$:

$$\overline{KB_2} \;=\; \frac{T_2}{2} \;=\; \frac{1.92}{2} \;=\; 0.96 \text{ m}$$

The position of the centre of gravity $\overline{KG_2}$ follows from the first moment of masses of the pontoon and the water ballast with respect to the base plane:

$$\Delta_2 \cdot \overline{KG_2} \;=\; \Delta_0 \cdot \overline{KG_0} \;+\; \Delta_{wb} \cdot z_{wb}$$

So:

$$\begin{aligned} \overline{KG_2} &= \frac{\Delta_0 \cdot \overline{KG_0} \;+\; \Delta_{wb} \cdot z_{wb}}{\Delta_2} \\ &= \frac{5022 \cdot 4.02 \;-\; 1215 \cdot 3.00}{5022 \;+\; 1215} \;=\; 3.82 \text{ m} \end{aligned}$$

Metacenter above centre of buoyancy $\overline{B_2M_2}$:

$$\begin{aligned}\overline{B_2M_2} &= \frac{I_T}{\nabla_2} = \frac{\frac{1}{12}LB^3}{LBT_2} = \frac{B^2}{12 \cdot T_2} \\ &= \frac{30.00^2}{12 \cdot 1.92} = 39.06 \text{ m}\end{aligned}$$

Herewith is the initial metacentric height of the pontoon with the 80 per cent filled tank with frozen water ballast $\overline{G_2M_2}$ known:

$$\begin{aligned}\overline{G_2M_2} &= \overline{KB_2} + \overline{B_2M_2} - \overline{KG_2} \\ &= 0.96 + 39.06 - 3.82 = 36.20 \text{ m}\end{aligned}$$

Now we let the water ballast unfreeze and the reduction $\overline{G_2G_2'}$ of the metacentric height becomes:

$$\begin{aligned}\overline{G_2G_2'} &= \frac{\rho_{wb} \cdot i_t}{\rho \cdot \nabla_2} \\ &= \frac{1.000 \cdot \frac{1}{12} \cdot 27.00 \cdot 7.50^3}{1.000 \cdot (5022 + 1215)} = 0.15 \text{ m}\end{aligned}$$

Herewith is the reduced initial metacentric height of the pontoon with the 80 per cent filled tank $\overline{G_2'M_2}$ known:

$$\begin{aligned}\overline{G_2'M_2} &= \overline{G_2M_2} - \overline{G_2G_2'} \\ &= 36.20 - 0.15 = 36.05 \text{ m}\end{aligned}$$

Add a heeling moment.
A transverse shift of the centre of gravity of the water ballast over a distance y_{wb} will result in a angle of heel ϕ_2.
Replace this shift by a heeling moment M_H:

$$M_H = \Delta_{wb} \cdot g \cdot y_{wb} \cdot \cos\phi_2$$

The righting stability moment of the pontoon M_S is:

$$M_S = \Delta_2 \cdot g \cdot \overline{G_2'M_2} \cdot \sin\phi_2$$

Because of the equilibrium $M_H = M_S$, it follows for the angle of heel ϕ_2:

$$\begin{aligned}\tan\phi_2 &= \frac{\Delta_{wb} \cdot y_{wb}}{\Delta_2 \cdot \overline{G_2'M_2}} \\ &= \frac{1215 \cdot 3.75}{(5022 + 1215) \cdot 36.05} \qquad \text{or: } \phi_2 = 1.16^0\end{aligned}$$

Note 1:
In this exercise, the effect of the free surface in the tank is very small.
In case of frozen cargo, the angle of heel ϕ_2 would be:

$$\begin{aligned}\tan\phi_2 &= \frac{\Delta_{wb} \cdot y_{wb}}{\Delta_2 \cdot \overline{G_2M_2}} \\ &= \frac{1215 \cdot 3.75}{(5022 + 1215) \cdot 36.20} \qquad \text{or: } \phi_2 = 1.15^0\end{aligned}$$

Note 2:
The small reduction of the metacentric height has been obtained in the design of the pontoon by a subdivision in the transverse direction in 4 compartments. As a result of this, the transverse moment of inertia (second moment of areas) of the free surface of the water ballast i_t has been reduced considerably.
With a tank over the full breadth, the reduction $\overline{G_2G_2'}$ of the metacentric height would be:

$$\begin{aligned}\overline{G_2G_2'} &= \frac{\rho_{wb} \cdot i_t}{\rho \cdot \nabla_2} \\ &= \frac{1.000 \cdot \frac{1}{12} \cdot 27.00 \cdot 30.00^3}{1.000 \cdot (5022 + 1215)} \;=\; 9.78 \text{ m}\end{aligned}$$

Solution of Part 3-e
Determine the initial metacentric height $\overline{GM}$ at an even keel draft of 7.50 meter, supposing that water will just not enter the deck.
Volume of displacement ∇ of pontoon:

$$\begin{aligned}\nabla &= L \cdot B \cdot T \\ &= 108.00 \cdot 30.00 \cdot 7.50 \;=\; 24300 \text{ m}^3\end{aligned}$$

Total mass of displacement Δ of pontoon:

$$\begin{aligned}\Delta &= \rho \cdot \nabla \\ &= 1.025 \cdot 24300 \;=\; 24908 \text{ ton}\end{aligned}$$

Mass of water ballast Δ_{wb} in pontoon:

$$\begin{aligned}\Delta_{wb} &= \Delta \;-\; \Delta_0 \\ &= 24980 \;-\; 5022 \;=\; 19886 \text{ ton}\end{aligned}$$

Height h of water level in ballast tanks:

$$\begin{aligned}h &= \frac{\Delta_{wb}}{\rho \cdot L \cdot B} \\ &= \frac{19886}{1.025 \cdot 108.00 \cdot 30.00} \;=\; 5.98 \text{ m}\end{aligned}$$

Centre of gravity of total water ballast with respect to half the length of the pontoon, the middle line plane and the base plane:

$$\begin{aligned}x_{wb} &= 0.00 \text{ m} \\ y_{wb} &= 0.00 \text{ m} \\ z_{wb} &= \frac{h}{2} = \frac{5.98}{2} \;=\; 2.99 \text{ m}\end{aligned}$$

For the ballasted pontoon, the position of the centre of gravity $\overline{KG}$ follows from the first moment of masses of the empty pontoon and the water ballast with respect to the base plane:

$$\Delta \cdot \overline{KG} \;=\; \Delta_0 \cdot \overline{KG_0} \;+\; \Delta_{wb} \cdot z_{wb}$$

So:

$$\begin{aligned}
\overline{KG} &= \frac{\Delta_0 \cdot \overline{KG_0} \ + \ \Delta_{wb} \cdot z_{wb}}{\Delta} \\
&= \frac{5022 \cdot 4.02 \ + \ 19886 \cdot 2.99}{24908} \ = \ 3.20 \text{ m}
\end{aligned}$$

Centre of buoyancy above keel $\overline{KB}$:

$$\overline{KB} \ = \ \frac{T}{2} \ = \ \frac{7.50}{2} \ = \ 3.75 \text{ m}$$

Metacenter above centre of buoyancy $\overline{BM}$:

$$\begin{aligned}
\overline{BM} &= \frac{I_T}{\nabla} = \frac{\frac{1}{12}LB^3}{LBT} = \frac{B^2}{12 \cdot T} \\
&= \frac{30.00^2}{12 \cdot 7.50} \ = \ 10.00 \text{ m}
\end{aligned}$$

The reduction of the metacentric height $\overline{GG'}$ due to the free surface of the water ballast becomes:

$$\begin{aligned}
\overline{GG'} &= \frac{\sum \rho_{wb} \cdot i_t}{\rho \nabla} \\
&= \frac{16 \cdot 1.025 \cdot \frac{1}{12} \cdot 27.00 \cdot 7.50^3}{24908} \ = \ 0.62 \text{ m}
\end{aligned}$$

Herewith is the reduced initial metacentric height of the pontoon $\overline{G'M}$ known:

$$\begin{aligned}
\overline{G'M} &= \overline{KB} + \overline{BM} - \overline{KG} - \overline{GG'} \\
&= 3.75 + 10.00 - 3.20 - 0.62 \ = \ 9.93 \text{ m}
\end{aligned}$$

Solution of Part 3-f
Determine the $\overline{GM}$-value at an even keel draft of 7.50 meter, supposing that water had just entered the deck.
With respect to Exercise 5, here $\overline{BM}$ changes only:

$$\overline{BM} \ = \ \frac{I_T}{\nabla}$$

The transverse moment of inertia (second moment of areas) of the water plane can be found by subtraction of moments of inertia of rectangles:

$$\begin{aligned}
I_T &= \frac{1}{12} \cdot \left\{ 9.00 \cdot \left(30.00^3 - 16.00^3 \right) + 18.00 \cdot \left(30.00^3 - 23.00^3 \right) \right\} \\
&= 39428 \text{ m}^4
\end{aligned}$$

or by using Steiner's rule:

$$\begin{aligned}
I_T &= 2 \cdot \left\{ \frac{1}{12} \cdot \left(9.00 \cdot 7.00^3 + 18.00 \cdot 3.50^3 \right) + 9.00 \cdot 7.00 \cdot 11.50^2 + 18.00 \cdot 3.50 \cdot 13.25^2 \right\} \\
&= 39428 \text{ m}^4
\end{aligned}$$

With this, the metacenter above centre of buoyancy $\overline{BM}$ becomes:

$$\overline{BM} \;=\; \frac{I_T}{\nabla} \;=\; \frac{39428}{24300} \;=\; 1.62 \text{ m}$$

Herewith is the reduced initial metacentric height of the pontoon $\overline{G'M}$ known:

$$\begin{aligned}\overline{G'M} \;&=\; \overline{KB} + \overline{BM} - \overline{KG} - \overline{GG'} \\ &=\; 3.75 + 1.62 - 3.20 - 0.62 \;=\; 1.55 \text{ m}\end{aligned}$$

Solution of Part 3-g
Determine the $\overline{GM}$-value, when the pontoon has been sink down until an even keel draft of 11.50 meter.
Volume of displacement ∇ of pontoon:

$$\nabla \;=\; 24300 + 2 \cdot (9.00 \cdot 7.00 + 18.00 \cdot 3.50) \cdot (11.50 - 7.50) \;=\; 25308 \text{ m}^3$$

Total mass of displacement Δ of pontoon:

$$\begin{aligned}\Delta \;&=\; \rho \cdot \nabla \\ &=\; 1.025 \cdot 25308 \;=\; 25941 \text{ ton}\end{aligned}$$

Mass of water ballast Δ_{wb} in pontoon:

$$\begin{aligned}\Delta_{wb} \;&=\; \Delta \;-\; \Delta_0 \\ &=\; 25941 \;-\; 5022 \;=\; 20919 \text{ ton}\end{aligned}$$

Height h of water level in ballast tanks:

$$\begin{aligned}h \;&=\; \frac{\Delta_{wb}}{\rho \cdot L \cdot B} \\ &=\; \frac{20919}{1.025 \cdot 108.00 \cdot 30.00} \;=\; 6.30 \text{ m}\end{aligned}$$

Centre of gravity of total water ballast with respect to half the length of the pontoon, the middle line plane and the base plane:

$$\begin{aligned}x_{wb} \;&=\; 0.00 \text{ m} \\ y_{wb} \;&=\; 0.00 \text{ m} \\ z_{wb} \;&=\; \frac{h}{2} \;=\; \frac{6.30}{2} \;=\; 3.15 \text{ m}\end{aligned}$$

For the ballasted pontoon, the position of the centre of gravity $\overline{KG}$ follows from the first moment of masses of the empty pontoon and the water ballast with respect to the base plane:

$$\Delta \cdot \overline{KG} \;=\; \Delta_0 \cdot \overline{KG_0} \;+\; \Delta_{wb} \cdot z_{wb}$$

So:

$$\begin{aligned}\overline{KG} \;&=\; \frac{\Delta_0 \cdot \overline{KG_0} \;+\; \Delta_{wb} \cdot z_{wb}}{\Delta} \\ &=\; \frac{5022 \cdot 4.02 \;+\; 20919 \cdot 3.15}{25941} \;=\; 3.32 \text{ m}\end{aligned}$$

Centre of buoyancy above keel $\overline{KB}$ follows from the first moment of volumes of the individual parts of the under water geometry of the pontoon with respect to the base plane:

$$\begin{aligned}\nabla \cdot \overline{KB} &= 108.00 \cdot 30.00 \cdot 7.50 \cdot 3.75 \\ &\quad +2 \cdot (9.00 \cdot 7.00 + 18.00 \cdot 3.50) \cdot (11.50 - 7.50) \cdot 9.50 \\ &= 100701 \text{ m}^4\end{aligned}$$

So:

$$\overline{KB} = \frac{100701}{25308} = 3.98 \text{ m}$$

Metacenter above centre of buoyancy $\overline{BM}$:

$$\overline{BM} = \frac{I_T}{\nabla} = \frac{39428}{25308} = 1.56 \text{ m}$$

The reduction of the metacentric height $\overline{GG'}$ due to the free surface of the water ballast becomes:

$$\begin{aligned}\overline{GG'} &= \frac{\sum \rho_{wb} \cdot i_t}{\rho \nabla} \\ &= \frac{16 \cdot 1.025 \cdot \frac{1}{12} \cdot 27.00 \cdot 7.50^3}{25941} = 0.60 \text{ m}\end{aligned}$$

Herewith is the reduced initial metacentric height of the pontoon $\overline{G'M}$ known:

$$\begin{aligned}\overline{G'M} &= \overline{KB} + \overline{BM} - \overline{KG} - \overline{GG'} \\ &= 3.98 + 1.56 - 3.32 - 0.60 = 1.62 \text{ m}\end{aligned}$$

Solution of Part 3-h
Determine the initial metacentric height of the rig.

$$\begin{aligned}\nabla_{rig} &= \frac{\Delta_{rig}}{\rho} = \frac{4920}{1.025} = 4800 \text{ m}^3 \\ T_{rig} &= \frac{\nabla_{rig}}{L_{rig} \cdot B_{rig}} = \frac{4800}{40.00 \cdot 40.00} = 3.00 \text{ m} \\ \overline{KB}_{rig} &= \frac{T_{rig}}{2} = \frac{3.00}{2} = 1.50 \text{ m} \\ \overline{BM}_{rig} &= \frac{I_{T_{rig}}}{\nabla_{rig}} = \frac{\frac{1}{12} L_{rig} \cdot B_{rig}^3}{L_{rig} \cdot B_{rig} \cdot T_{rig}} = \frac{B_{rig}^2}{12 \cdot T_{rig}} = \frac{40.00^2}{12 \cdot 3.00} = 44.44 \text{ m} \\ \overline{GM}_{rig} &= \overline{KB}_{rig} + \overline{BM}_{rig} - \overline{KG}_{rig} = 1.50 + 44.44 - 20.00 = 25.94 \text{ m}\end{aligned}$$

Solution of Part 3-i
Determine the initial metacentric height of pontoon+rig, supposing that they just hit each other when inflating the pontoon during loading the rig (centre of rig above centre of pontoon).
Draft T at even keel of pontoon:

$$T = 7.50 + 3.00 = 10.50 \text{ m}$$

Volume of displacement ∇ of pontoon:

$$\nabla \;=\; 24300 + 2 \cdot (9.00 \cdot 7.00 + 18.00 \cdot 3.50) \cdot (10.50 - 7.50) \;=\; 25056 \text{ m}^3$$

Total mass of displacement Δ of pontoon:

$$\begin{aligned} \Delta &= \rho \cdot \nabla \\ &= 1.025 \cdot 25056 \;=\; 25683 \text{ ton} \end{aligned}$$

Mass of water ballast Δ_{wb} in pontoon:

$$\begin{aligned} \Delta_{wb} &= \Delta \;-\; \Delta_0 \\ &= 25683 \;-\; 5022 \;=\; 20661 \text{ ton} \end{aligned}$$

Height h of water level in ballast tanks:

$$\begin{aligned} h &= \frac{\Delta_{wb}}{\rho \cdot L \cdot B} \\ &= \frac{20661}{1.025 \cdot 108.00 \cdot 30.00} \;=\; 6.22 \text{ m} \end{aligned}$$

Centre of gravity of total water ballast with respect to half the length of the pontoon, the middle line plane and the base plane:

$$\begin{aligned} x_{wb} &= 0.00 \text{ m} \\ y_{wb} &= 0.00 \text{ m} \\ z_{wb} &= \frac{h}{2} \;=\; \frac{6.22}{2} \;=\; 3.11 \text{ m} \end{aligned}$$

For the ballasted pontoon + rig, the position of the centre of gravity $\overline{KG}$ follows from the first moment of masses of the empty pontoon, the water ballast and the rig with respect to the base plane of the pontoon:

$$(\Delta + \Delta_{rig}) \cdot \overline{KG} \;=\; \Delta_0 \cdot \overline{KG_0} \;+\; \Delta_{wb} \cdot z_{wb} \;+\; \Delta_{rig} \cdot z_{rig}$$

With $z_{rig} = 7.50 + 20.00 = 27.50$ meter, it is found:

$$\begin{aligned} \overline{KG} &= \frac{\Delta_0 \cdot \overline{KG_0} \;+\; \Delta_{wb} \cdot z_{wb} \;+\; \Delta_{rig} \cdot z_{rig}}{\Delta + \Delta_{rig}} \\ &= \frac{5022 \cdot 4.02 \;+\; 20661 \cdot 3.11 \;+\; 4920 \cdot 27.50}{25683 + 4920} \;=\; 7.18 \text{ m} \end{aligned}$$

The centre of buoyancy above keel $\overline{KB}$ follows from the first moment of volumes of the individual parts of the under water geometry of the pontoon and the rig with respect to the base plane of the pontoon:

$$\begin{aligned} (\nabla + \nabla_{rig}) \cdot \overline{KB} \;=\; & \; 108.00 \cdot 30.00 \cdot 7.50 \cdot 3.75 \\ & +2 \cdot (9.00 \cdot 7.00 + 18.00 \cdot 3.50) \cdot 3.00 \cdot 9.00 \\ & +4800 \cdot (7.50 + 1.50) \\ =\; & \; 141129 \text{ m}^4 \end{aligned}$$

So:

$$\overline{KB} = \frac{141129}{25056 + 4800} = 4.73 \text{ m}$$

Metacenter above centre of buoyancy $\overline{BM}$:

$$\overline{BM} = \frac{I_T}{\nabla + \nabla_{rig}} = \frac{39428 + 40.00 \cdot 40.00^3}{25056 + 4800} = 8.47 \text{ m}$$

The reduction of the metacentric height $\overline{GG'}$ due to the free surface of the water ballast becomes:

$$\begin{aligned} \overline{GG'} &= \frac{\sum \rho_{wb} \cdot i_t}{\rho \nabla} \\ &= \frac{16 \cdot 1.025 \cdot \frac{1}{12} \cdot 27.00 \cdot 7.50^3}{25056 + 4800} = 0.51 \text{ m} \end{aligned}$$

Herewith is the reduced initial metacentric height of the pontoon $\overline{G'M}$ known:

$$\begin{aligned} \overline{G'M} &= \overline{KB} + \overline{BM} - \overline{KG} - \overline{GG'} \\ &= 4.73 + 8.47 - 7.18 - 0.51 = 5.51 \text{ m} \end{aligned}$$

Solution of Part 3-j

Determine the initial metacentric height of pontoon+rig when all water ballast has been removed from the pontoon.

$$\begin{aligned} \Delta &= \Delta_0 + \Delta_{rig} = 5022 + 4920 = 9942 \text{ ton} \\ \nabla &= \frac{\Delta}{\rho} = \frac{9942}{1.025} = 9700 \text{ m}^3 \\ T &= \frac{\nabla}{L \cdot B} = \frac{9700}{108.00 \cdot 30.00} = 3.00 \text{ m} \\ \overline{KB} &= \frac{T}{2} = \frac{3.00}{2} = 1.50 \text{ m} \\ \overline{BM} &= \frac{I_T}{\nabla} = \frac{B^2}{12 \cdot T} = \frac{30.00^2}{12 \cdot 3.00} = 25.00 \text{ m} \\ \overline{KG} &= \frac{5022 \cdot 4.02 + 4920 \cdot 27.50}{5022 + 4920} = 15.64 \text{ m} \\ \overline{GG'} &= 0.00 \text{ m} \\ \overline{GM} &= \overline{KB} + \overline{BM} - \overline{KG} = 1.50 + 25.00 - 15.64 = 10.86 \text{ m} \end{aligned}$$

Solution of Part 3-k

Determine the angle of heel when, due to an inaccurate loading of the pontoon, the centre of the rig is amidships 1.0 meter outside the middle line plane of the pontoon.

$$\begin{aligned} M_H &= 4920 \cdot g \cdot 1.00 \cdot \cos\phi \\ M_S &= (5022 + 4920) \cdot g \cdot 10.86 \cdot \sin\phi \\ \tan\phi &= \frac{4920 \cdot 1.00}{(5022 + 4920) \cdot 10.86} = 0.0456 \qquad \text{or: } \phi = 2.61^0 \end{aligned}$$