Reprinted: 01-11-2000
Revised: 03-10-2007
Website: www.shipmotions.nl

Report 440, September 1976,
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Motions and Resistance of a Ship in Regular Following Waves

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Summary

In continuation of experiments in head waves an experimental program has been set up to measure vertical motions and added resistance in regular following waves. The results of these experiments are compared with theoretical calculations.

1 List of Symbols

z	Instantaneous wave elevation
z_a	Wave amplitude
z_w	Wave height (double amplitude)
z	Heave displacement
z_a	Heave amplitude
q	Pitch angle
q_a	Pitch amplitude
e	Phase angle
w	Circular frequency
w_e	Circular frequency of encounter
m	Wave direction (following waves
l	Wave length
$k = 2p / l$	Wave number
r	Density of water
g	Acceleration of gravity
L	Length between perpendiculars
B	Breadth
c	Wave celerity

V	Forward speed
$Fn = V / \sqrt{gL}$	Froude number
$N^{'}$	Sectional damping coefficient for zero speed
$m^{'}$	Sectional added mass
V_{za}	Amplitude of vertical relative water velocity
x, y, z	Right-handed co-ordinate system, moving with the ship
x_b, y_b, z_b	Right-handed co-ordinate system, fixed to the ship
R_{AW}	Added resistance due to waves
s	Relative vertical motion
s_a	Amplitude of relative vertical motion
s_m	Crest or trough value of relative vertical motion

2 Introduction

To continue a research program with a model of a fast cargo ship in regular head waves, see Gerritsma and Beukelman (1972) and Journée (1976), a test program has been set up to measure vertical motions and resistance in regular following waves.
The results of these experiments at several speeds and wavelengths have been compared with computer calculations. These have been carried out with a program named TRIAL, of which Beukelman and Bijlsma (1973) have described an earlier version. Only heave and pitch motions are observed. The added mass and damping for the ship cross sections have been calculated by using a Lewis conformal transformation.
Apart from heave and pitch motions the relative vertical motions of the after body at station 0 and station 2 are measured. By comparing the directly measured relative motions with those calculated from the measured heave, pitch and wave motions, the dynamical rise of the water surface near the hull at these two stations can be estimated.
At the same time the total resistance has been measured. By subtracting the still water resistance from this total resistance, the added resistance due to waves has been found.

3 Description of Model and Experiments

Fast cargo ship m.s. "S.A. van der Stel" was designed for lifting and transporting cargo items in excess of 100 tons, with an additional accommodation for 12 passengers. The vessel was designed and built for South African Marine Corporation Ltd. in Cape Town, South Africa. She was launched at 26 February 1966 at Verolme Shipyards in Heusden, the Netherlands. The ship, with a length of 152.50 meter, has a service speed of 19.5 knots.

m.s. S.A. van der Stel

The experiments have been carried out with a 3-meter model of this ship in full load condition. The model scale is 1:50.
The main particulars are given in Table 1.

L or L_{pp}	(m)	3.050
B	(m)	0.456
d at even keel	(m)	0.183
∇	(m^3)	0.1434
C_b	(-)	0.564
L_{cb} / L_{pp}	(%)	-1.10
k_{yy} / L_{pp}	(%)	21.88

Table 1 Main Particulars of Model

Figure 1 and Figure 2 show the body plan and the experimental set-up for these tests.
In case of head waves the influence of the surge motion on heave and pitch motions can usually be neglected, as demonstrated by Gerritsma and Beukelman (1972) and Journée (1976). During the experiments in regular following waves, described here, the model has no freedom to surge for practical reasons. Especially at lower periods of encounter it is very difficult to produce a constant towing force or a constant propulsion force which corresponds with the mean total resistance of the model. Also tank wall interference will be of more importance at a surging model in a relatively narrow tank. The surging model introduces wave, disturbances which make it impossible to measure the undisturbed wave motion.
Five speeds in following waves have been considered corresponding with the Froude numbers $Fn = 0.00$, 0.15, 0.20, 0.25 and 0.30, while the service speed of the ship corresponds with $Fn = 0.26$.
Heave and pitch motions have been measured by two low friction potentiometers at the centre of gravity of the model.
A conductance wave probe close to the hull has measured the relative motions with respect to the water surface of the aft part of the model at station 0 and 2. The regular waves have been

measured in the same way by means of a two-wire conductive wave probe which position with respect to the model is shown in Figure 2.
An ultra-violet recorder records all motions.
The still water resistance and the average resistance in waves have been measured by means of a strain gauge dynamometer of which the output is integrated over a certain time or a full number of wave periods.
To get information about the influence of a rotating propeller, the vertical motions are measured too at a speed corresponding with $Fn = 0.25$ with a self-propelled model restrained in surge. Because the added resistance due to waves is very small, the number of revolutions per second of the propeller has in this case been obtained from propulsion tests in still water. This number was kept constant by electronic control.

4 Analysis and presentation of the test results

The definitions and symbols of the vertical motions are shown in Figure 3.
The wave, heave and pitch motions are defined by:

Wave: $z = z_a \cdot \cos(w_e t)$

Heave: $z = z_a \cdot \cos(w_e t + e_{zz})$

Pitch: $q = q_a \cdot \cos(w_e t + e_{qz})$

where the frequency of encounter in deep water is defined by:

$$w_e = w - \frac{w^2}{g} \cdot V \cdot \cos m$$

From this equation the relation between the frequency of encounter and the wave length-ship length ratio can be found as shown in Figure 4. In following waves m will be zero degrees.

At a certain speed two types of waves can be distinguished here:

1. The model is overtaking the waves, which means:

$$\frac{l}{L} < 2p \cdot (Fn \cdot \cos m)^2$$

2. The waves are overtaking the model, which means:

$$\frac{l}{L} > 2p \cdot (Fn \cdot \cos m)^2$$

All experiments have been carried out in following waves, which were overtaking the model. For these waves the frequency of encounter has a maximum attainable value:

$$w_{e_{\max}} = \frac{0.25\sqrt{\frac{g}{L}}}{Fn \cdot \cos m} = \frac{g}{4 \cdot V \cdot \cos m}$$

at:

$$\frac{l}{L} = 8p \cdot \left(Fn \cdot \cos m\right)^2$$

It can be found from reference Hanaoka (1933) that in the towing tank used wall interference will occur if:

$$w_e \cdot Fn \cdot \cos m < 0.43\sqrt{\frac{g}{L}}$$

However the maximum attainable value is:

$$w_e \cdot Fn \cdot \cos m = 0.25\sqrt{\frac{g}{L}}$$

This means that it is not possible to do experiments with the model considered in this towing tank without tank wall interference. During the experiments it appeared that wall effects became important at low frequencies of encounter, when the wave celerity decreased to the model speed.

4.1 Heave and Pitch Motions

The linear relation between heave and wave amplitude at each observed speed and wavelength is shown in Figure 5. It may be noticed that this linearity is very good. Figure 6 shows that the wave amplitude does not influence the phase lag between heave and wave motion. In Figure 7 and Figure 8, the same is demonstrated for the pitch motion.
Experiments at $Fn = 0.25$ with a self-propelled model restrained in surge show that the influence of a rotating propeller on the heave and pitch motions could be neglected. This is also demonstrated in Figure 5, Figure 6, Figure 7 and Figure 8.
The average non-dimensional measured heave and pitch amplitudes, obtained by the least squares method, and the mean phase lags are compared with the results of calculations made with the program TRIAL, mentioned before. The good agreement is shown in Figure 9 and Figure 10.

4.2 Relative Motions

Because of potential flow effects due to the speed the effective freeboard in still water differs from the geometrical freeboard. Figure 11 shows the measured relative displacement in still water at station 0 and 2 as a function of the model speed.
In regular waves dynamic phenomena will influence the immersion amplitude. The measured crest and trough values of the relative motions aft are presented in Figure 12-a-b-c-d-e versus the wave amplitude for different wavelengths and speeds. The experiments with a self-propelled model restrained in surge are shown here too. Besides a good linearity the experiments show that the influence of a rotating propeller on the relative motions of the after body can be neglected.
The slopes of these lines, calculated with the least square method, are shown in Figure 13 versus the wave length-ship length ratio for all speeds. It may be noticed that, just like for the

relative motions at the fore body in head waves, see Journée (1976), the average measured crest and trough values are not the same.
The undisturbed relative motions have been calculated from the measured heave, pitch and wave motions. The amplitudes are presented in Figure 13 too. There is a fair agreement with the values calculated by means of the program TRIAL. The differences between the directly measured relative motions and the relative motions calculated from the measured heave, pitch and wave motions are caused by dynamic phenomena. In contradiction with the tests in head waves, this dynamic phenomenon is not only depending on the undisturbed relative motion and the speed but also on the frequency of encounter.

4.3 Added Resistance due to Waves

The added resistance in following waves is obtained by subtracting the still water resistance from the measured total resistance at the speed concerned. It appears that the added resistance in following waves is positive and very small. It varies as the squared wave amplitude. This is shown in Figure 14.
The mean non-dimensional values of the added resistance, obtained by the least square method, are presented in Figure 15.
Gerritsma and Beukelman (1972) have introduced a method to calculate the added resistance of a ship in longitudinal waves. This method is based on the determination of the radiated energy of the damping waves. Neglecting the surge motion, heave pitch motions have been calculated with the strip theory. Added mass and damping for the ship cross sections have been calculated by using a Lewis conformal transformation. In this way the added resistance will be:

$$R_{AW} = \frac{k}{2 \cdot w_e} \cdot \int_0^L \left(N' - V \cdot \frac{dm'}{dx_b} \right) \cdot V_{za}^{\ 2} \cdot dx_b$$

As shown by Gerritsma and Beukelman (1972) and Journée (1976), there is a fair agreement between theory and experiments in case of head waves. In following waves the added resistance has been calculated in the same way. The results in Figure 16 show a considerable negative added resistance. The significant deviation from the experiments will be investigated in the near future.

5 Conclusions

From the analysis of the experiments and calculations for a model, restricted in surge in regular following waves it may be concluded:

1. At each wave length and speed:
   - The amplitude of heave and pitch motions varies linearly as the wave amplitude.
   - Phase differences between heave and pitch motions and wave motions are constant for varying wave amplitudes.
   - Crest and trough values of the relative motions of the after body vary linearly as the wave amplitude.
   - Added resistance varies as the square of the wave amplitude.
2. A rotating propeller does not influence the heave and pitch motions and the relative motions of the after body of a model restricted in surge.
3. Calculated heave and pitch motions are in very good agreement with the measurements.

4. The procedure to predict the added resistance in following waves needs further investigation.
5. Dynamic phenomena are of importance when calculating the relative motions of the after body.

6 Acknowledgement

The assistance of Mr. A.J. van Strien during the experiments and the preparation of the graphs by Mr. P.W. de Heer are gratefully acknowledged.

7 References

Beukelman and Bijlsma (1973)
W. Beukelman and E.F. Bijlsma, *Description of a Program to Calculate the Behaviour of a Ship in a Seaway (named TRIAL)*, Delft Shiphydromechanics Laboratory, Report 383, August 1973.

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J. Gerritsma and W. Beukelman, *Analysis of the Resistance Increase in Waves of a Fast Cargo Ship*, Netherlands Ship Research Centre, Report 169S, April 1979.

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T. Hanaoka, *On the Velocity Potential in Mitchell's System and the Configuration of the Wave Ridges due to a Moving Ship*, Japanese Society of Naval Architects, Report 93, 1933.

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8 Figures

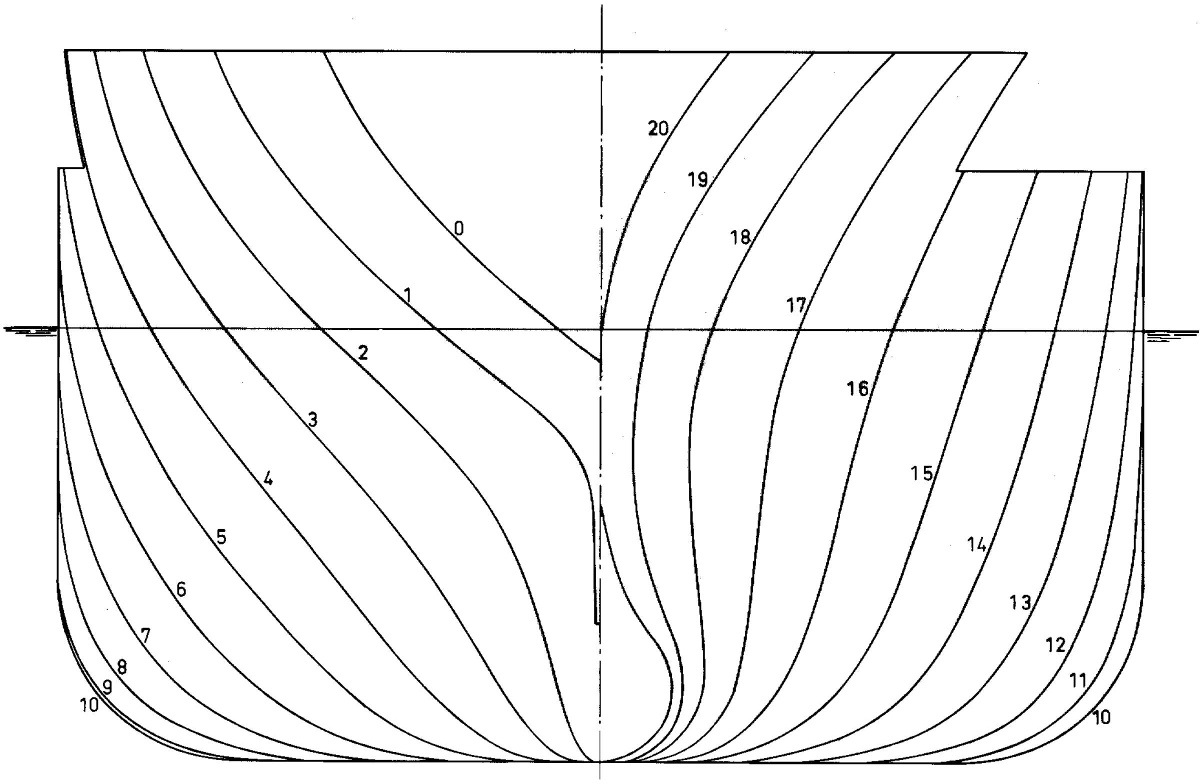

Figure 1 Body Plan

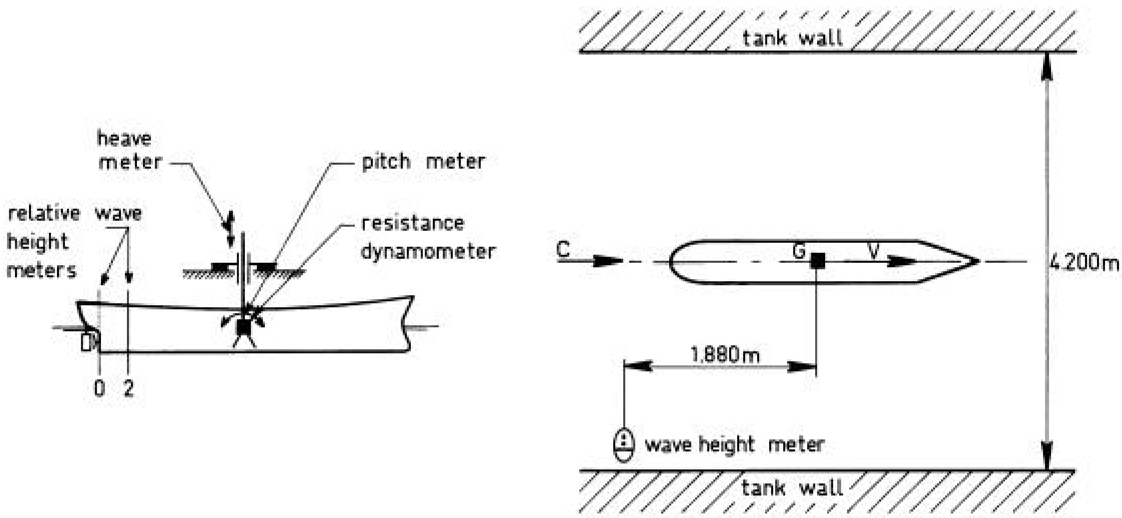

Figure 2 Experimental Set-Up

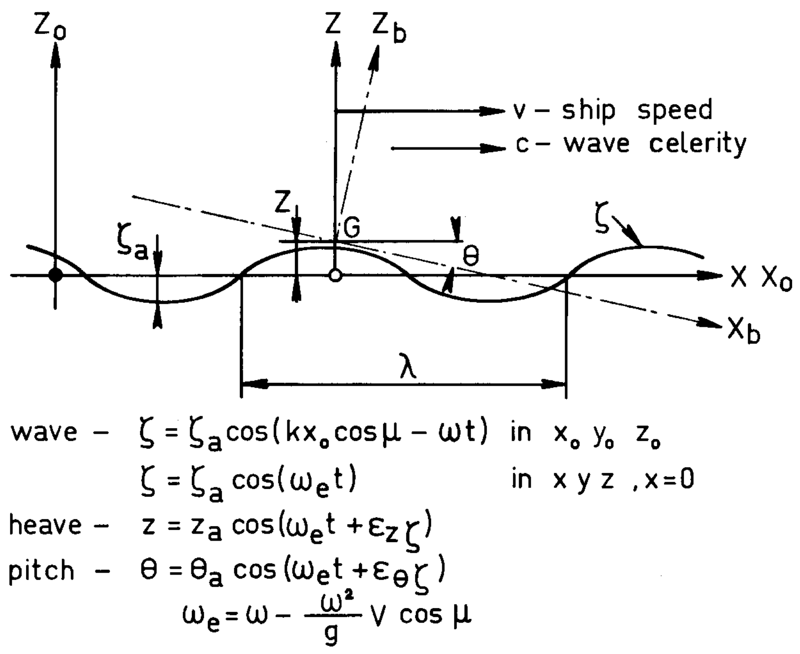

Figure 3 Symbols and Definitions

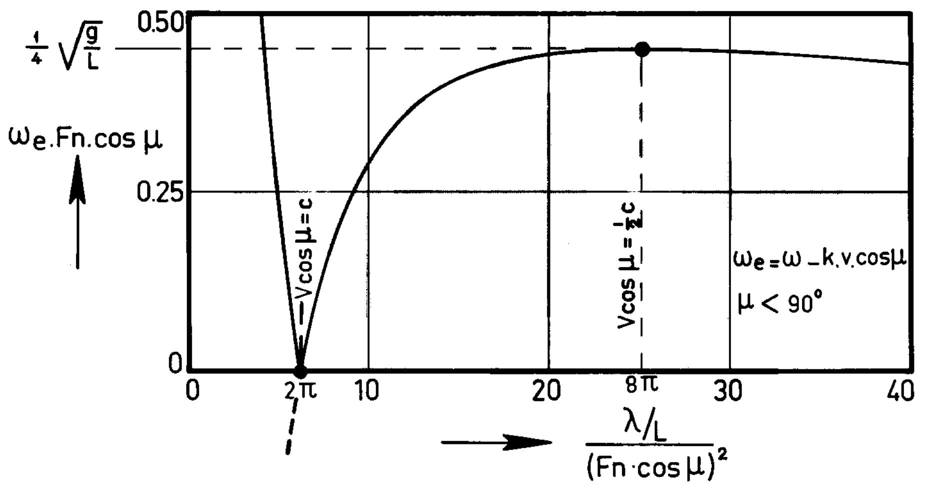

Figure 4 Relation between Frequency of Encounter and Wave Length

in Following Waves

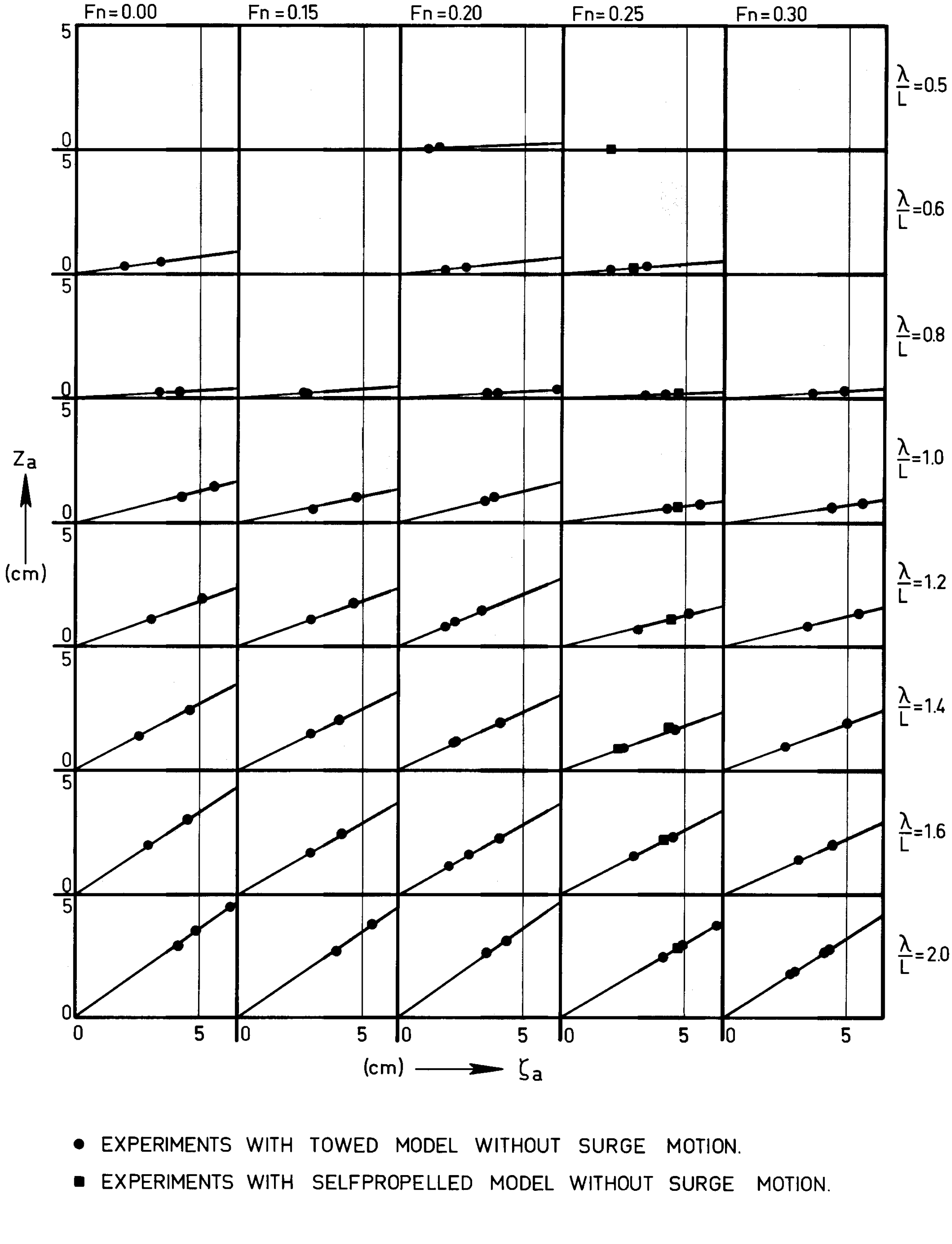

Figure 5 Relation between Heave and Wave Amplitude

in Following Waves

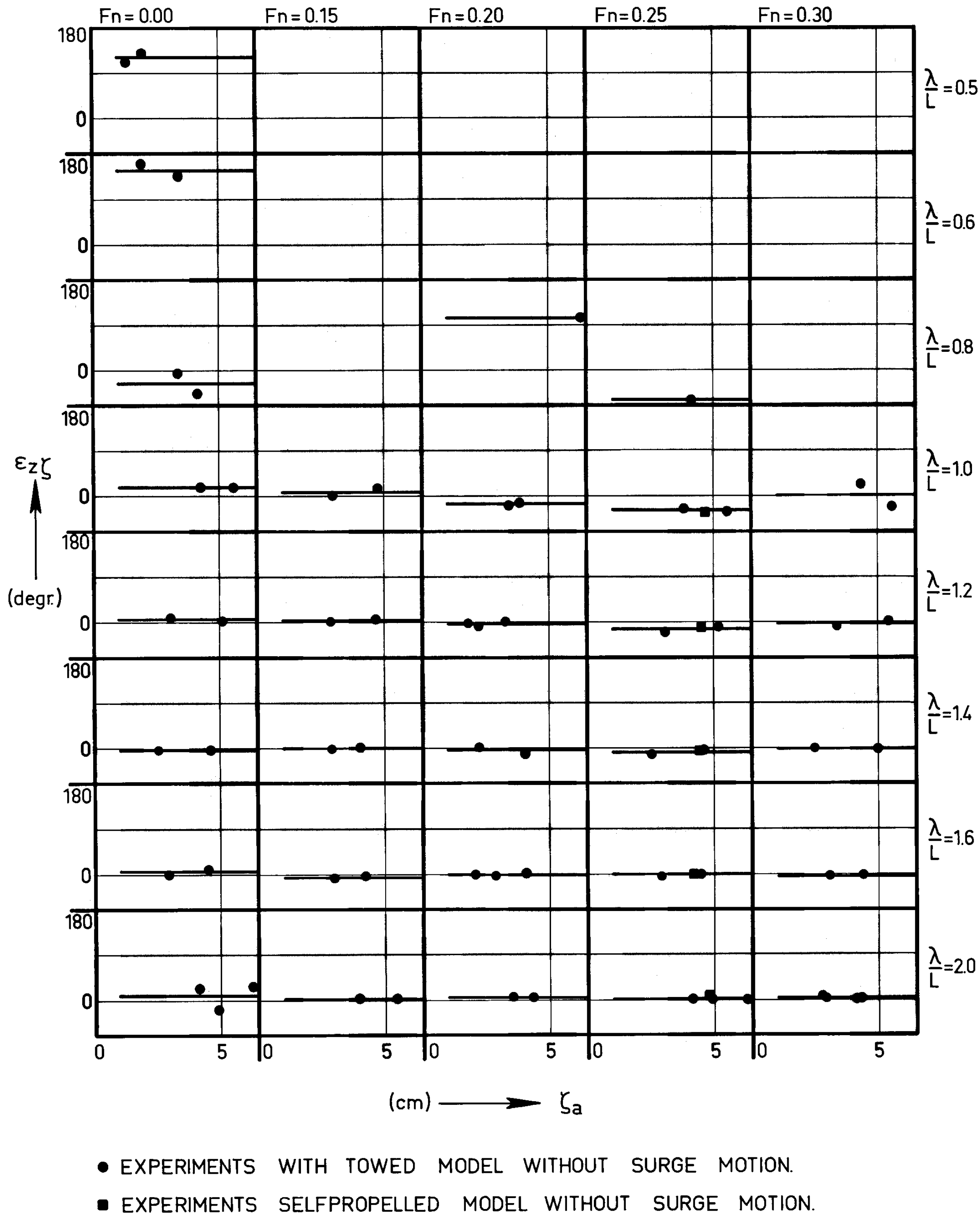

Figure 6 Relation between Phase Lag of Heave and Wave and Wave Amplitude in Following Waves

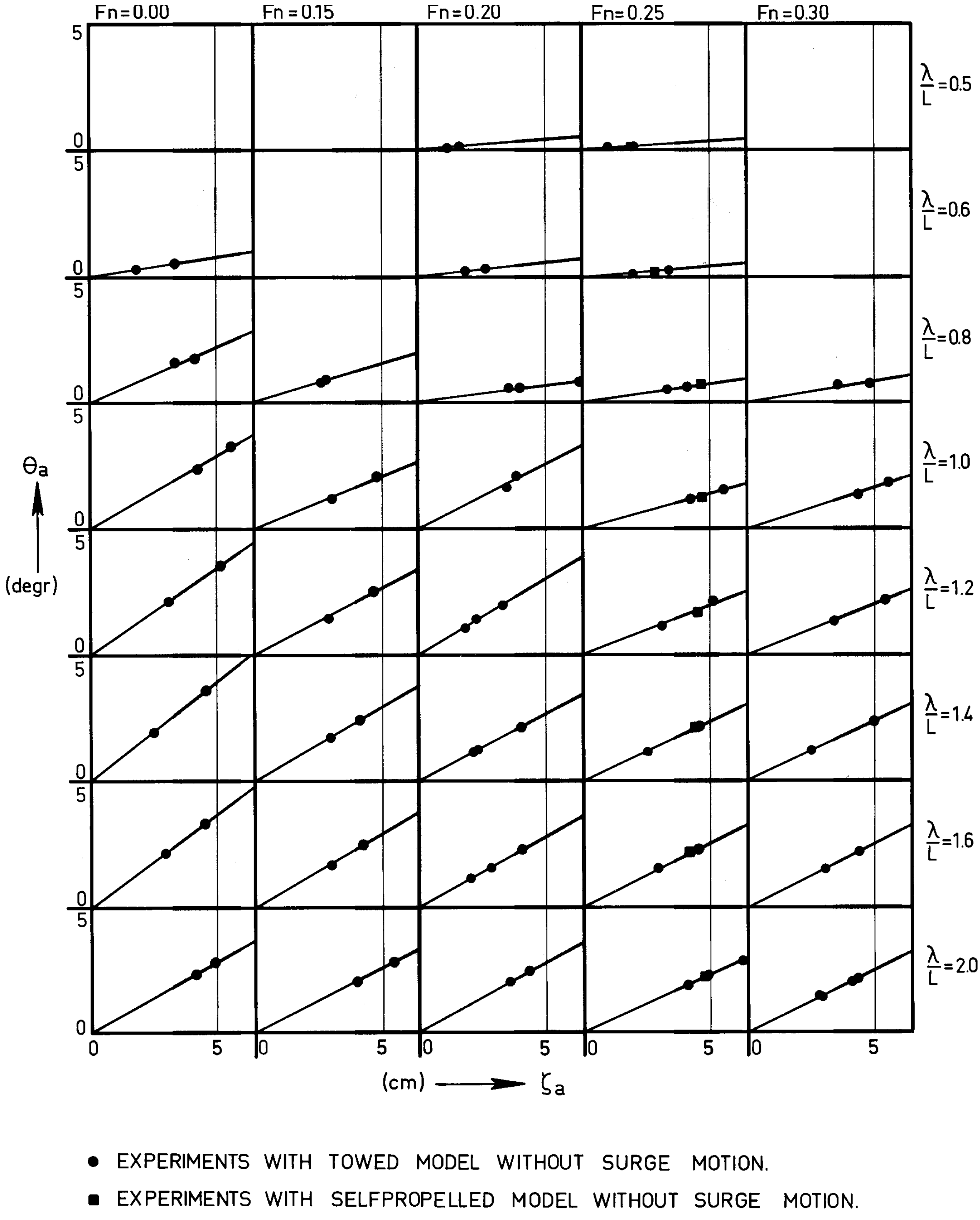

Figure 7 Relation between Pitch and Wave Amplitude in Following Waves

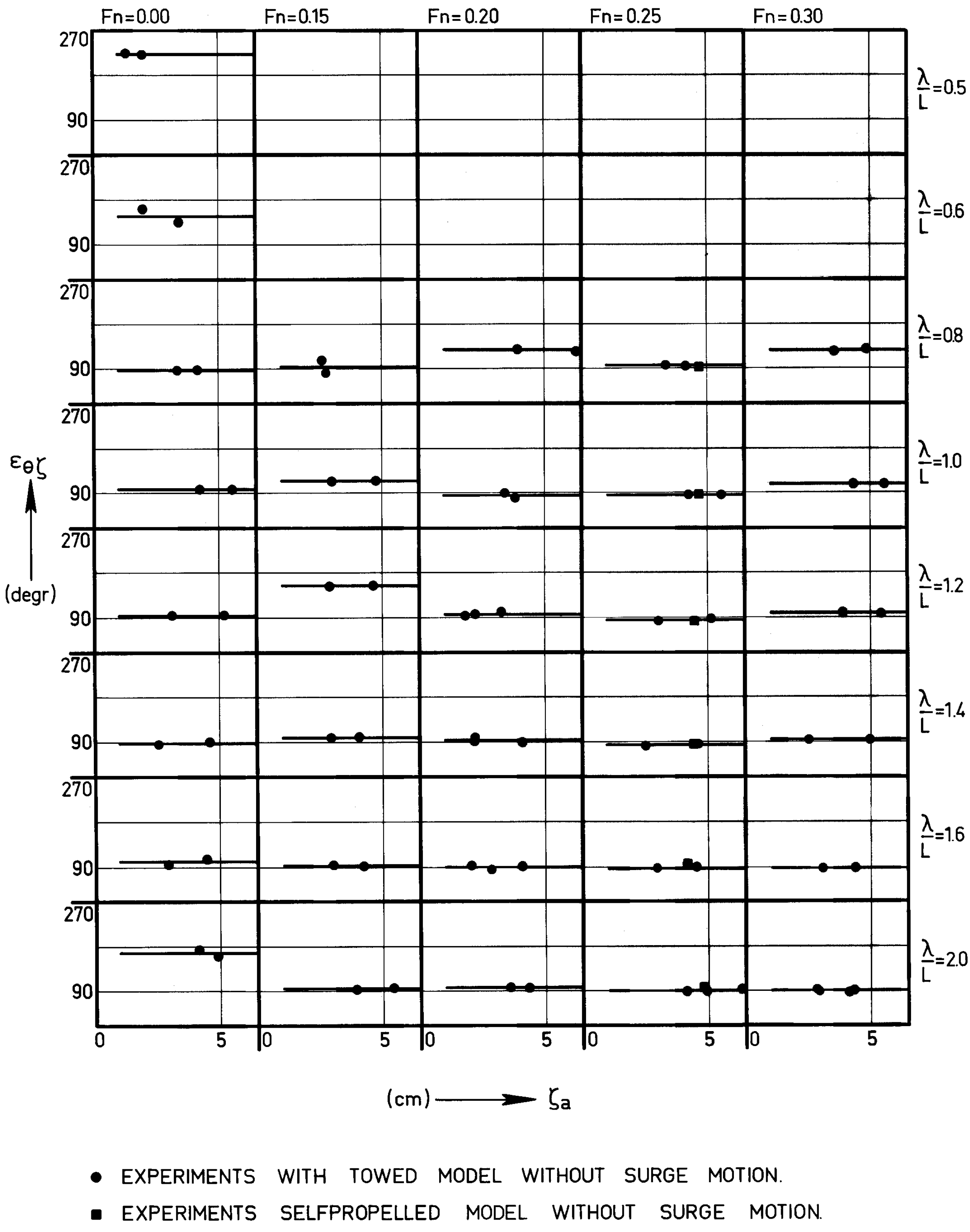

Figure 8 Relation between Phase Lag of Pitch and Wave and Wave Amplitude in Following Waves

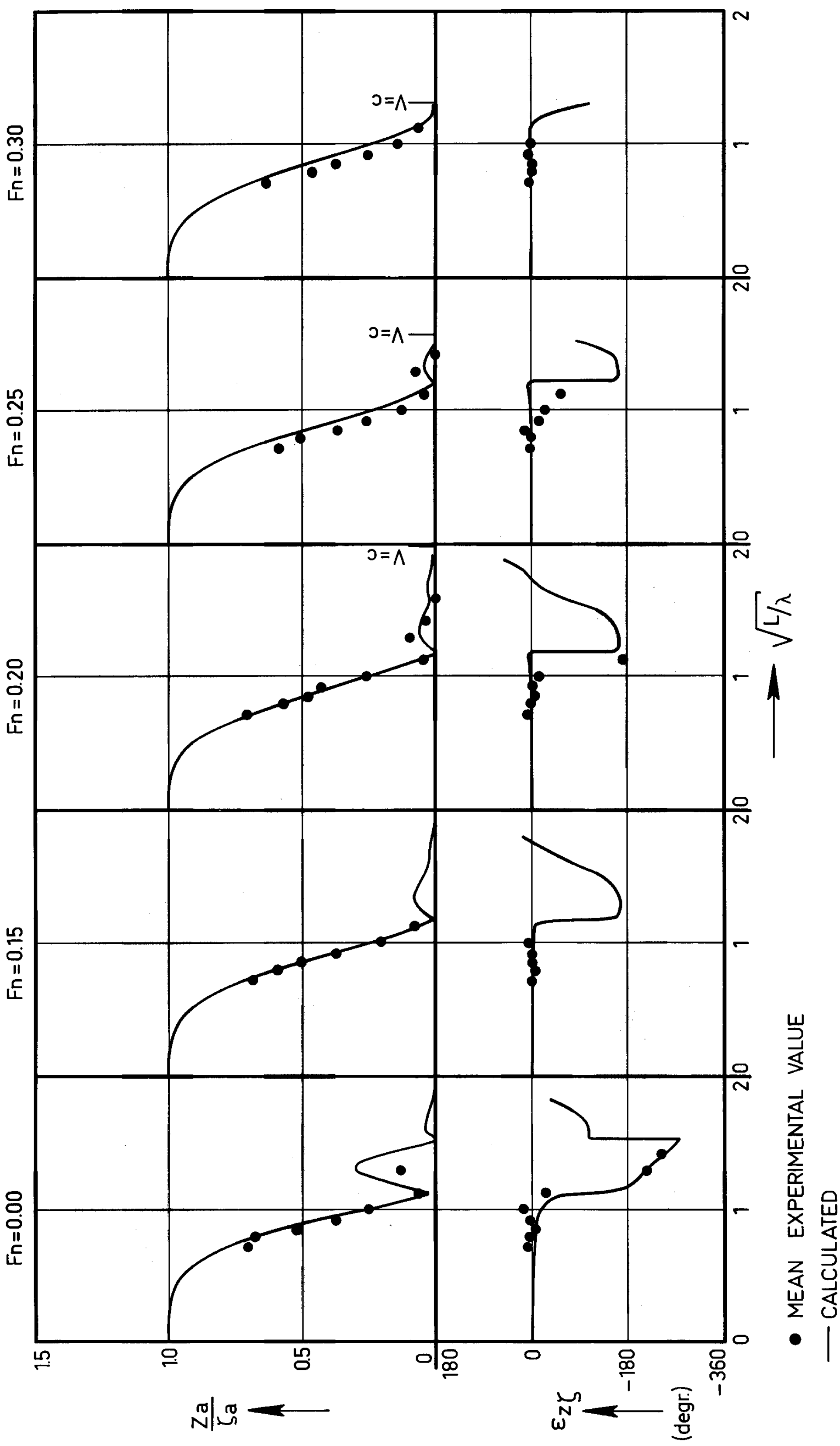

Figure 9 Measured and Calculated Characteristics of Heave Motions in Following Waves

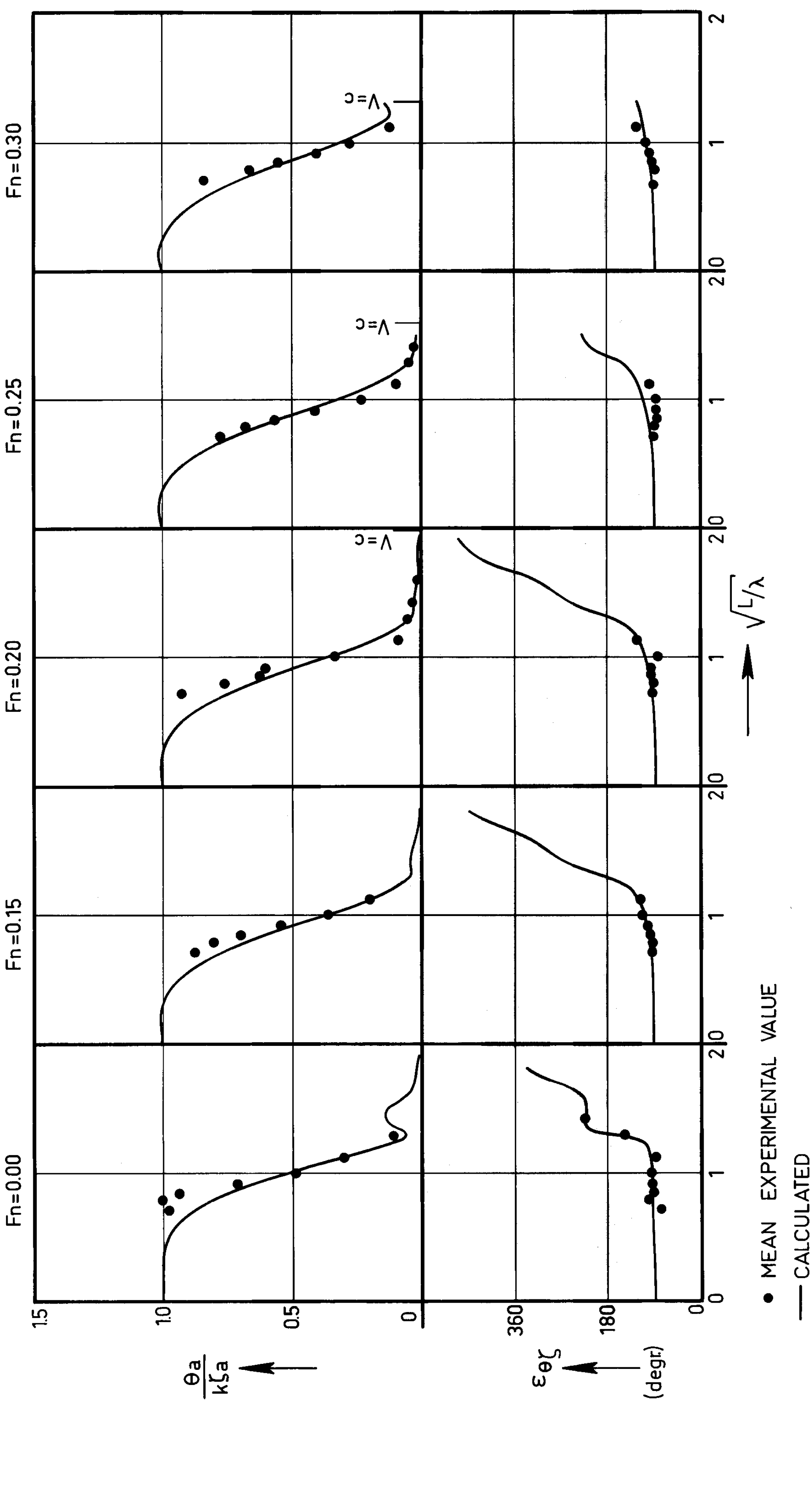

Figure 10 Measured and Calculated Characteristics of Pitch Motions

in Following Waves

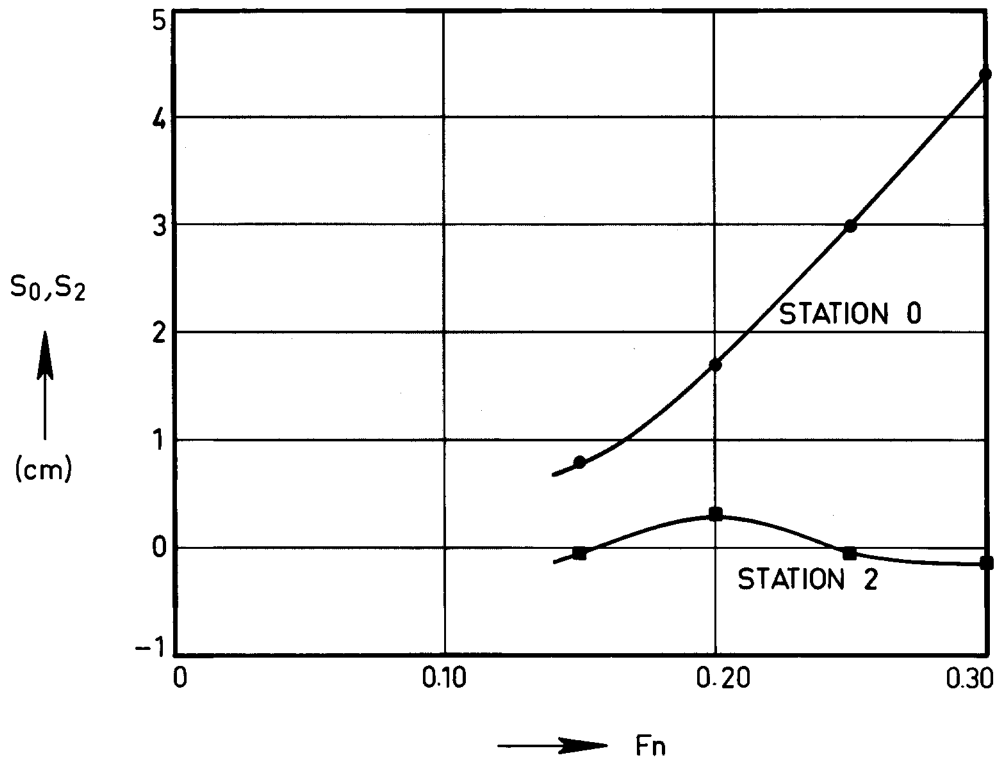

Figure 11 Relative Displacement of the After Body in Still Water

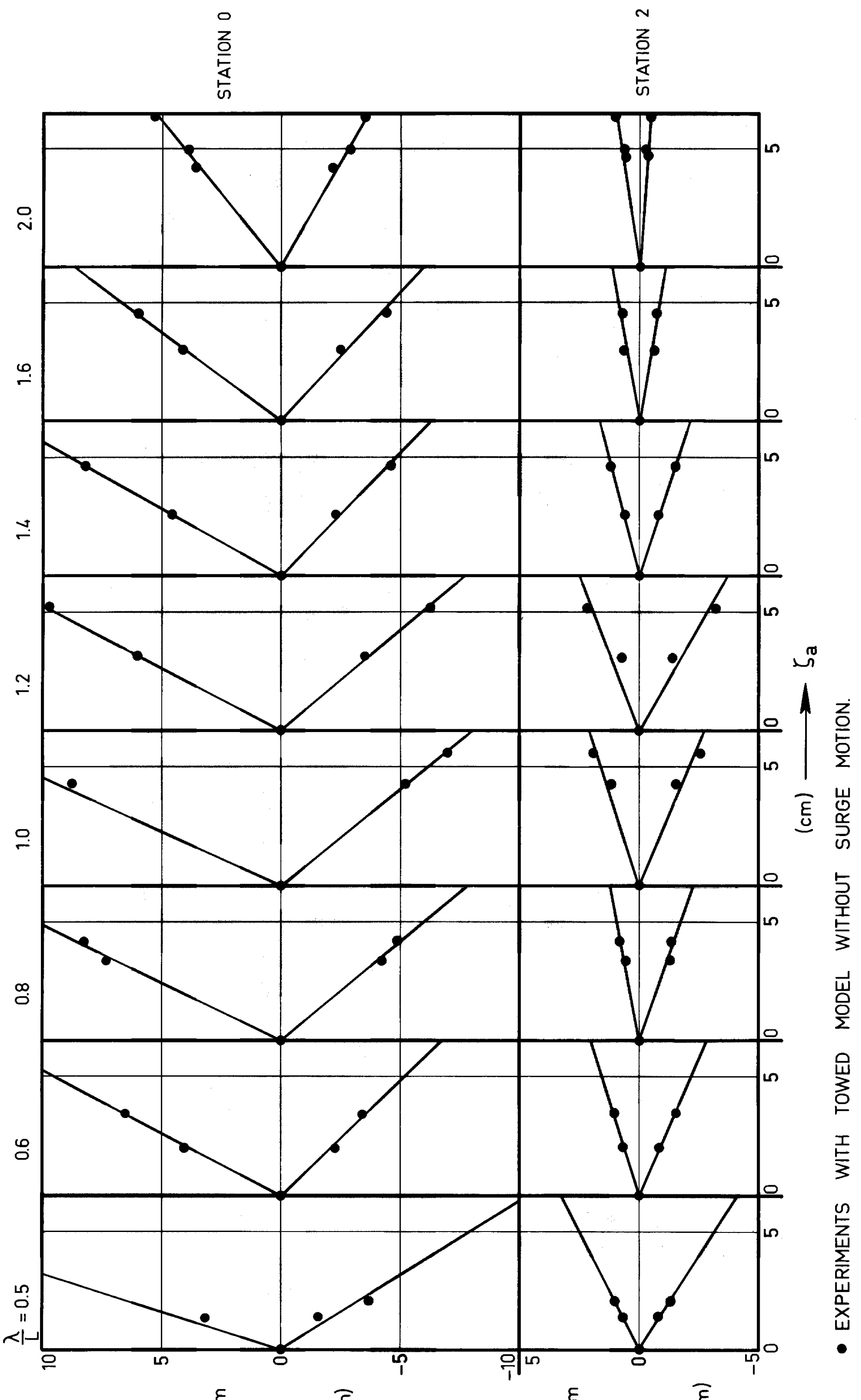

Figure 12–a Relation between Crest and Through Values of Relative Motions and Wave Amplitudes in Following Waves (Fn = 0.00)

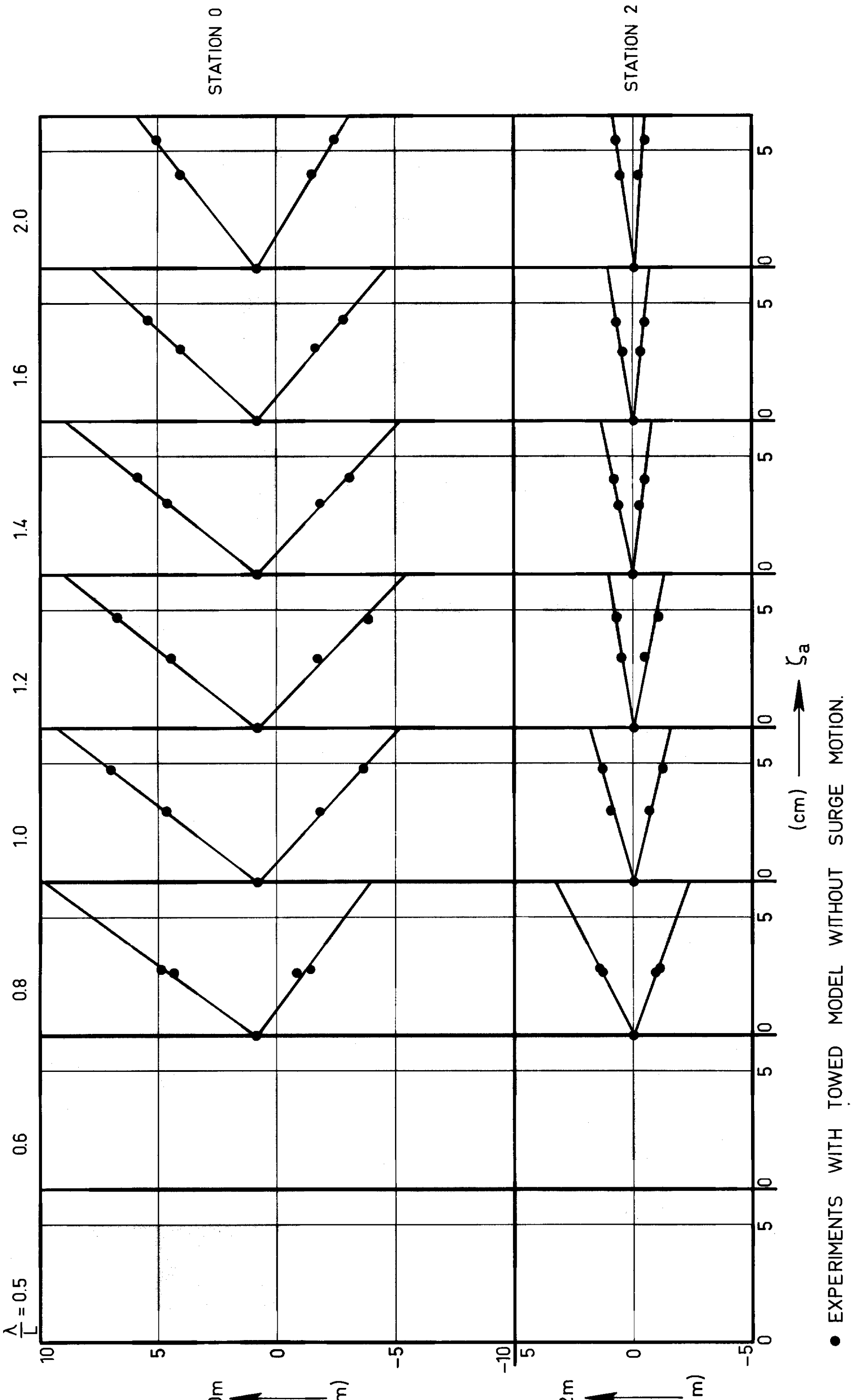

Figure 12-b Relation between Crest and Through Values of Relative Motions and Wave Amplitudes in Following Waves (Fn = 0.15)

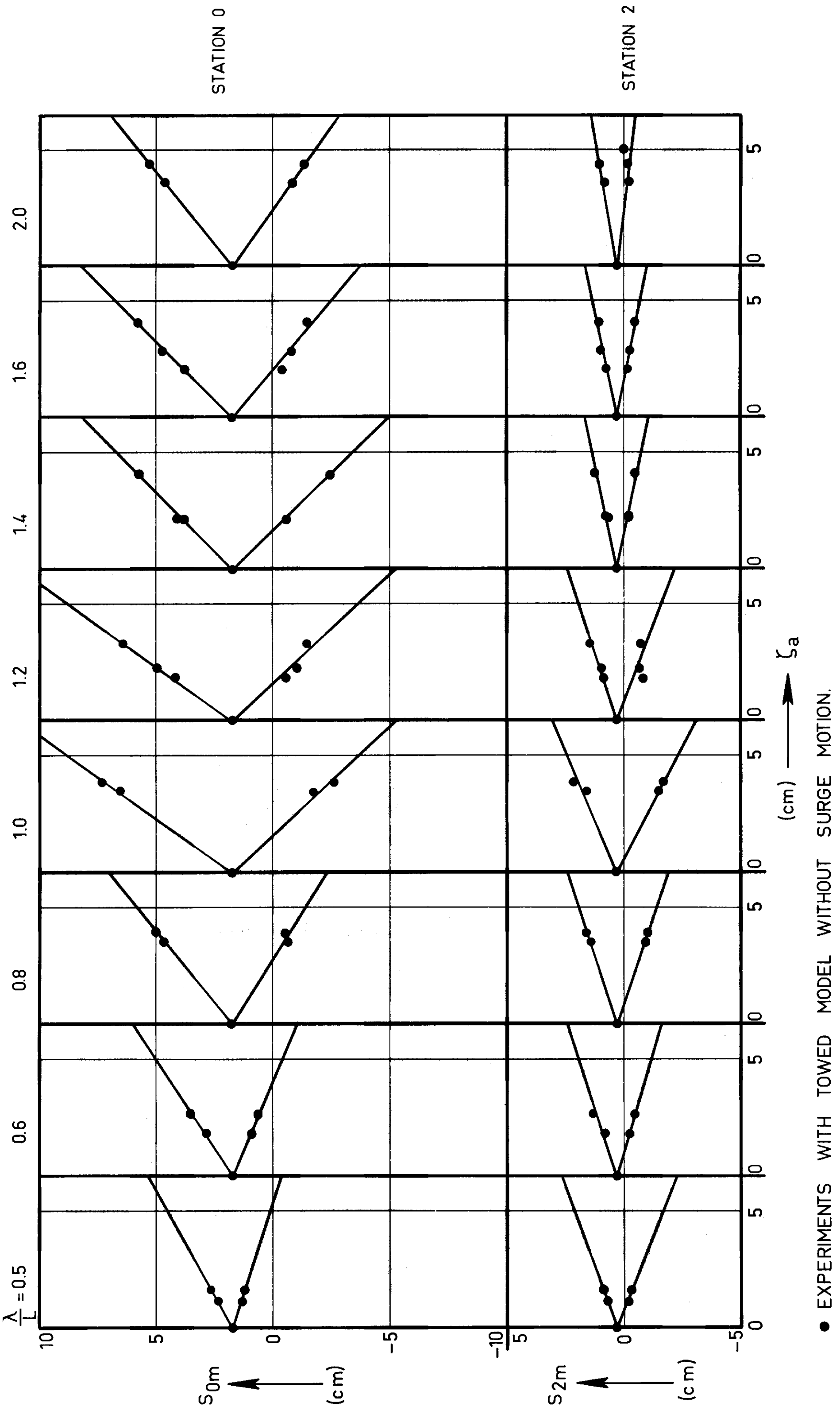

Figure 12-c Relation between Crest and Through Values of Relative Motions and Wave Amplitudes in Following Waves (Fn = 0.20)

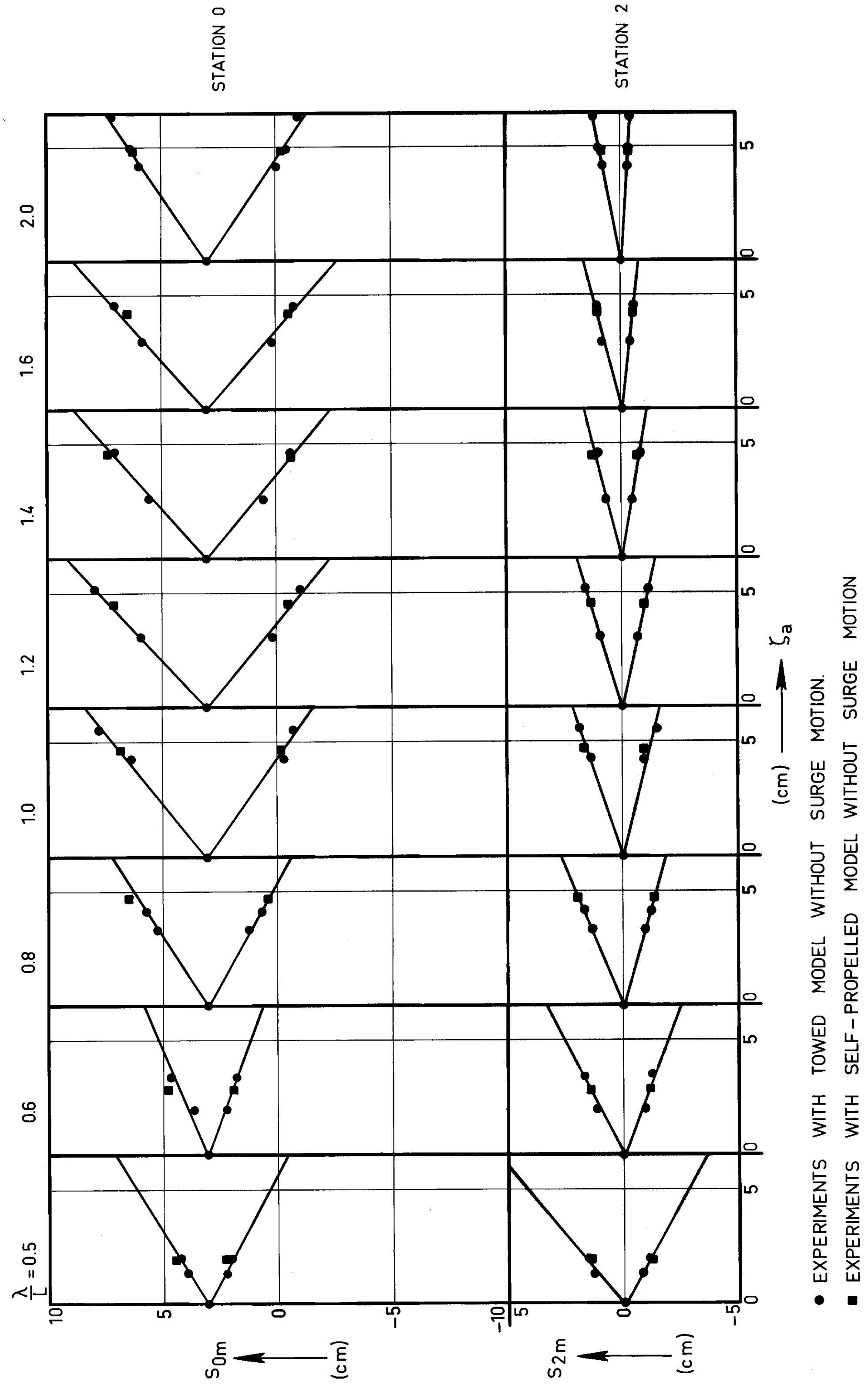

Figure 12-d Relation between Crest and Through Values of Relative Motions and Wave Amplitudes in Following Waves (Fn = 0.00)

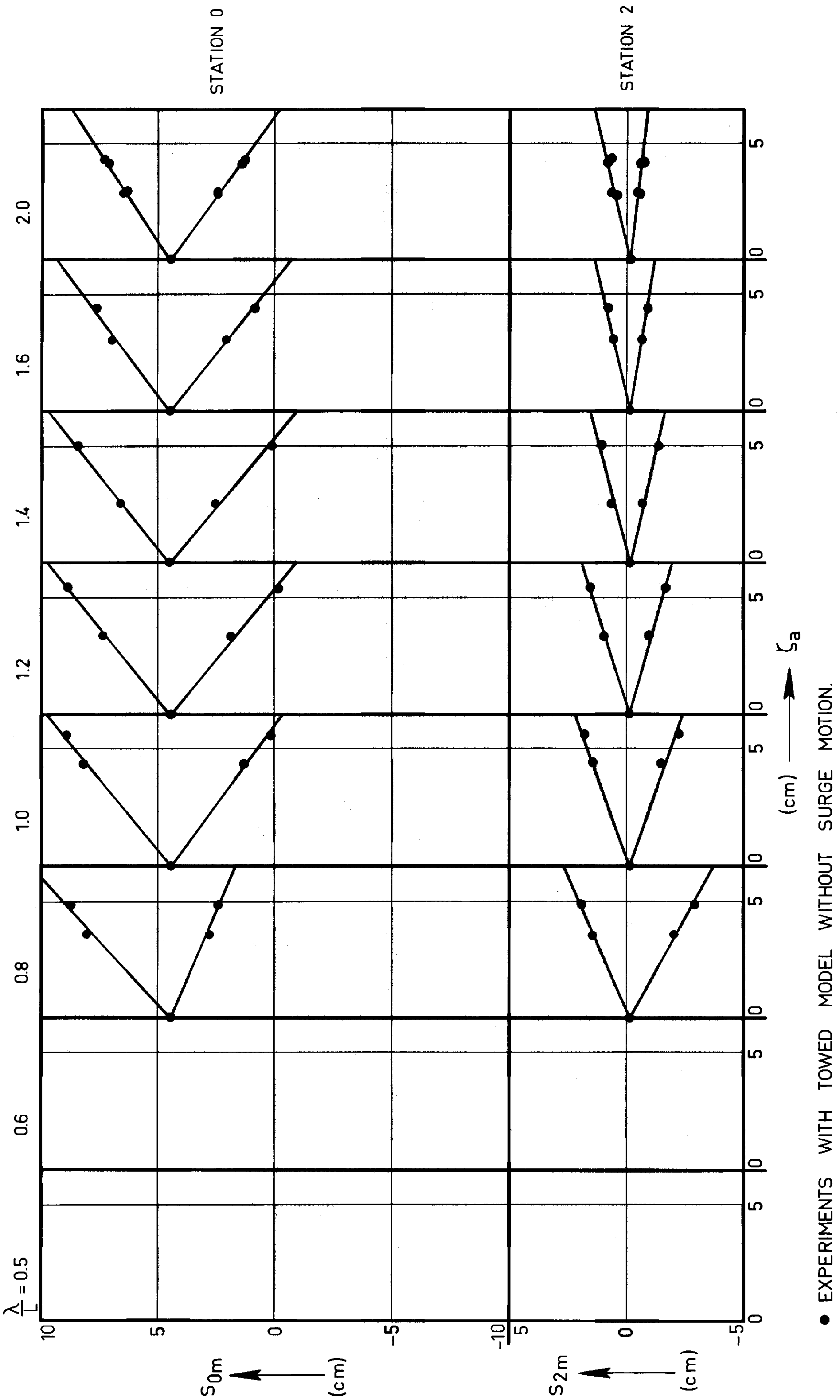

Figure 12-e Relation between Crest and Through Values of Relative Motions and Wave Amplitudes in Following Waves (Fn = 0.30)

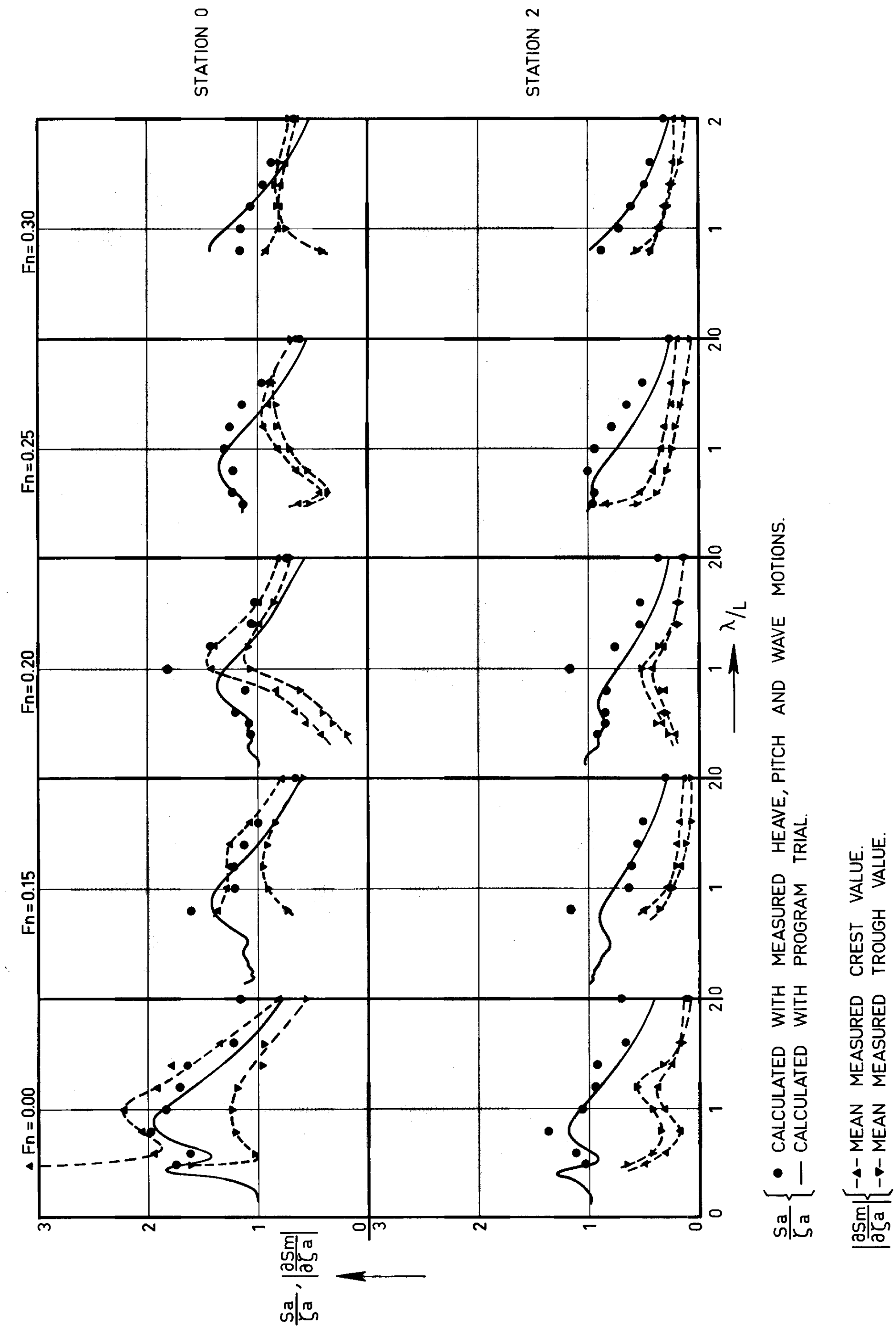

Figure 13 Measured and Calculated Characteristics of Relative Motions

in Following Waves

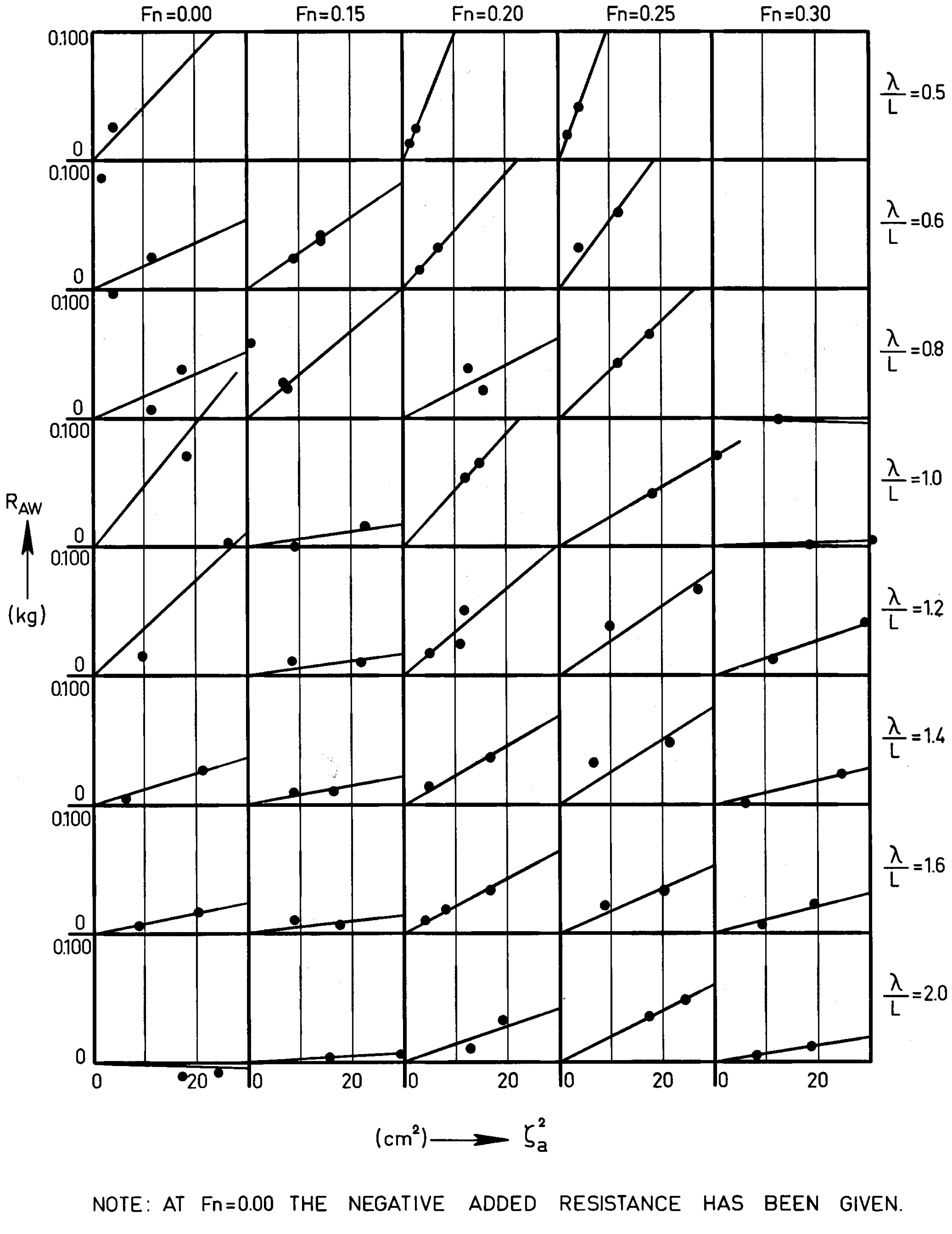

Figure 14 Relation between Added resistance and Wave Amplitude Squared

in Following Waves

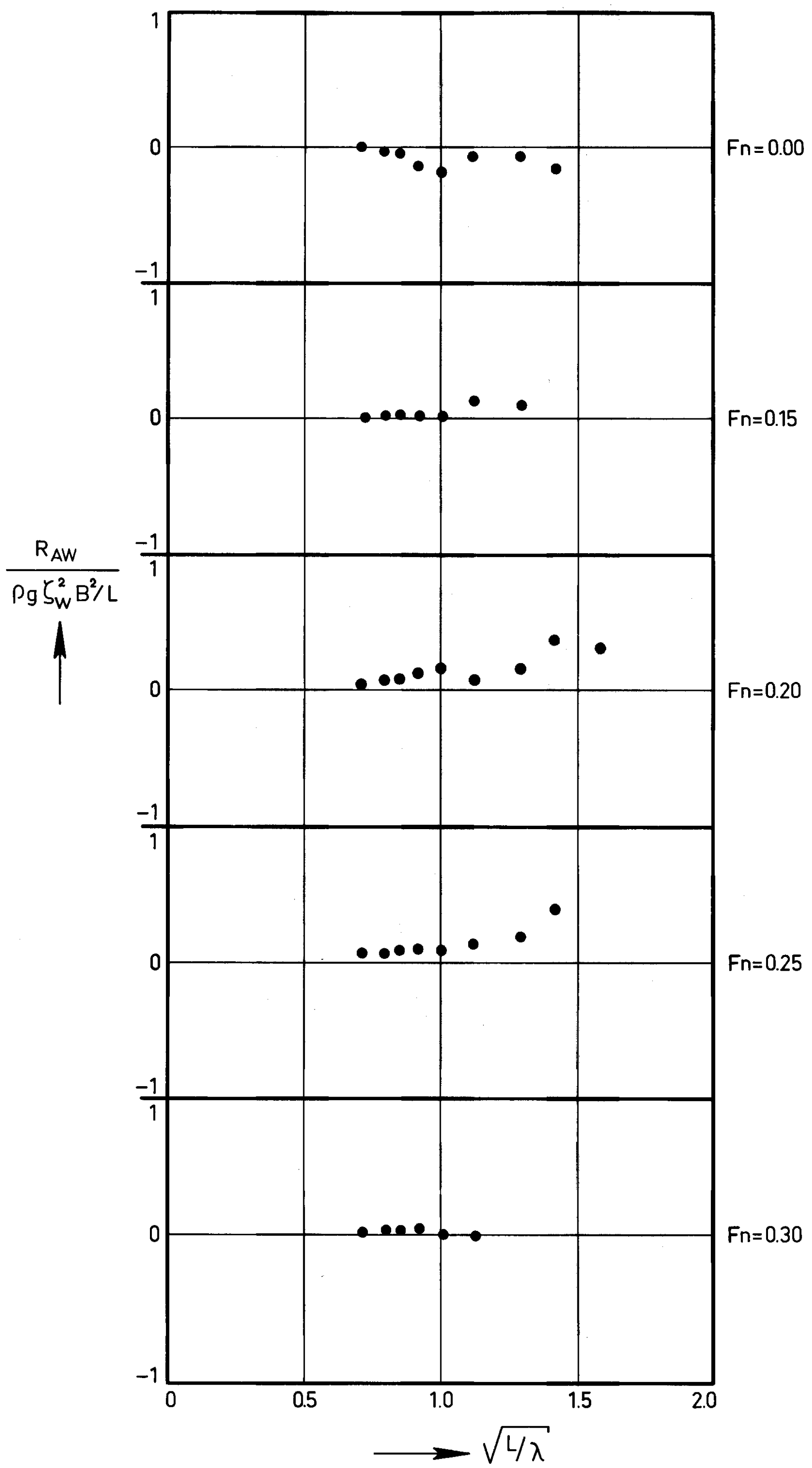

Figure 15 Non-Dimensional Measured Added Resistance in Following Waves

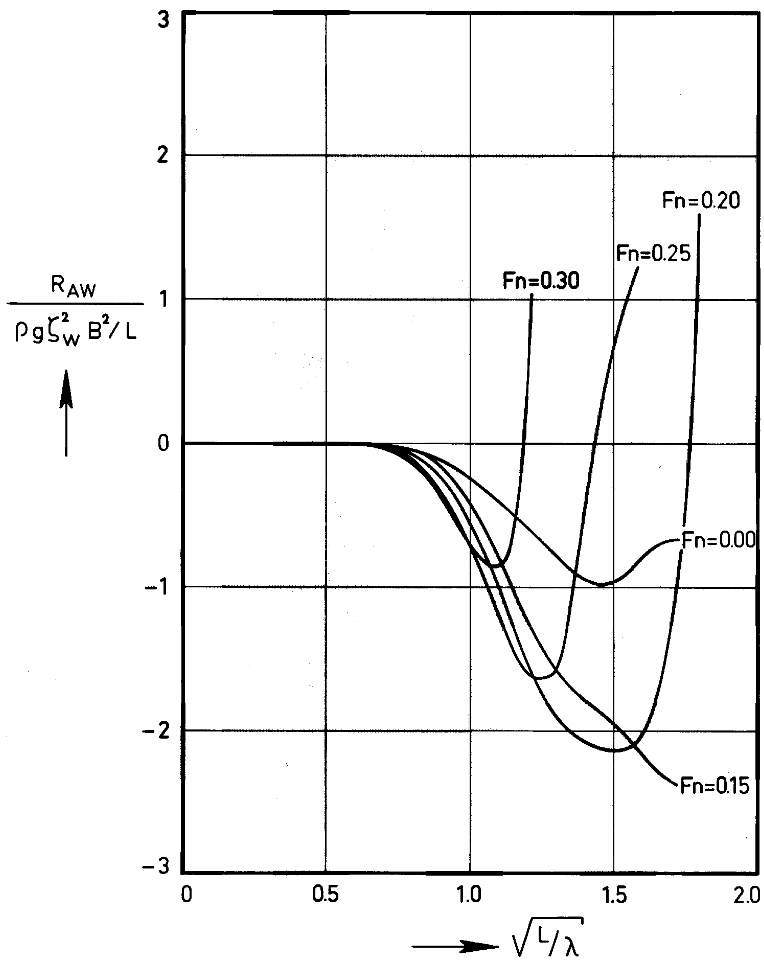

Figure 16 Non-Dimensional Calculated Added Resistance in Following Waves