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First Order Wave Loads in Beam Waves

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Abstract

In many cases, the simple strip-theory method with the classic relative motion approach delivers a poor prediction of the first order wave loads for sway, heave and roll of a cross-section of a ship in beam waves.
A better prediction is given by the diffraction theory, which calculates the diffracted wave system by using Green's second identity and the known incident wave potential. A disadvantage of this method is that the wave load calculations are relatively time-consuming, because these calculations have to be repeated for each wave direction.
Therefore, a simple calculation method to obtain the wave loads, without solving the diffraction problem itself for each wave direction, would be welcome. As a first attempt for beam waves only, this paper presents a very quick, simple and accurate method to obtain the wave loads from the potential coefficients and the radiated wave energy. Results of this method have been compared with computational results of the diffraction theory and a perfect fit in beam waves was found.
Also the radiated damping waves, produced by the oscillating ship, have been used to account for the amplification of the incoming wave system. Calculated vertical relative motions have been compared with experimental data and a fair agreement was found.

1 Introduction

If an incident wave system encounters a body, a diffracted wave system will be induced by the presence of the body. The disturbed flow pattern causes a pressure distribution on the body and will therefore introduce a hydrodynamic force, the diffraction force.

The diffracted wave system can be calculated by applying Green's second theorem and using the known incident wave potential. A method to calculate the two-dimensional potentials, describing the diffraction problem, was first presented by Frank (1967), and is generally known as the "Frank Close-Fit" method. Another method, based on Lewis conformal mapping coefficients, is given by Keil

(1974). These two and some other methods are implemented in the strip theory computer program SEAWAY by Journée (1992a), used here to obtain the calculated data. The advantage of the direct diffraction calculations is that the amplitudes and phases of the loads are obtained without any difficulty, because the in and out phase parts of the loads are calculated. The disadvantage is that the calculation speed of the strip theory program is slowed down, since the calculations are complex and the diffraction problem has to be solved for each wave direction.

In the classic relative motion theory, sectional averaged orbital motions of the water particles are calculated from the pressure distribution in the undisturbed waves on the section contour. With these average orbital accelerations and velocities and the potential mass and damping coefficients, the in and out phase parts of the diffraction loads are calculated. The advantage of using the relative motion approach is that the amplitudes and phases of the loads are obtained from the potential coefficients, which are independent of the wave direction. This method delivers a fairly accurate prediction of the 2-D first order wave loads for heave of a ship sailing in bow waves. But in beam waves the method gives in many cases a poor prediction of the wave loads for sway, heave and roll, when compared with results when using the diffraction theory.

With the Haskind-Newman method, see Newman (1962), the amplitudes of the overall wave loads on a cross section of a ship can be obtained from the potential damping coefficients, without solving the diffraction problem itself. This relationship can be derived analytically from the radiated energy of the cross section. However, no information concerning the phases can be obtained.

In this paper, the phases of the wave loads with respect to the incoming waves are obtained from the results of the potential mass and damping calculations. A perfect fit with results of diffraction calculations was found.

In the classic strip-theory with the relative motion approach, the amplification of the incoming waves, due to waves produced by the oscillating ship, has not been accounted for. Based on the radiated wave energy approach, a very simple method is given here to calculate this amplification, often referred to as dynamic swell-up. Vertical relative motions in regular head waves, measured at the bow of a model of a cargo ship with forward ship speed, are compared with calculated data. A fair agreement was found.

2 2-D Coordinate System

A right handed orthogonal axes system has been used here with the vertical x_{3b}-axis at the centerline of the cross section, positive upwards, and the lateral x_{2b}-axis in the waterline, positive to the right.

Roll motions and roll moments are positive when turning from x_{2b} to x_{3b}. The wave direction μ is zero in following waves. Beam waves with $\mu = 90^0$ are travelling in the positive x_{2b} direction.

In the calculations, two typical different cross-sections of a 200,000 TDW crude oil tanker are used. The shape of these two cross sections is presented in Figure 1.

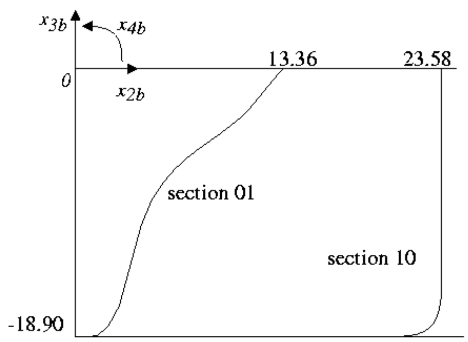

Figure 1 Cross Sections 01 and 10 of a 200,000 TDW Crude Oil Carrier

Cross section 01 has a heeled side wall and cross section 10 has a vertical side wall at the intersection with the waterline.

3 Potential Coefficients

Suppose an infinite long cylinder in the still water surface of a fluid. The cylinder is forced to carry out a simple harmonic motion about its initial position with frequency of oscillation w and small amplitude of displacement x_{2a} for sway, x_{3a} for heave and x_{4a} for roll:

$$x_j = x_{ja} \cdot \cos wt \quad \text{for:} \quad j = 2, 3, 4$$

Equation 1

The 2-D hydrodynamic loads X_{hi} in the sway, heave or roll directions i, exercised by the fluid on a cross section of the cylinder, can be obtained from the 2-D velocity potentials and the linearised equation of Bernoulli. The velocity potentials have been obtained by using an N-parameter conformal mapping method of the cross section to the unit circle.
The hydrodynamic loads are defined by:

$$X_{hi} = 2 r y_{wl} \frac{g z_{ja}}{p} \cdot \cdot \left(A_{ij} \cos\left(wt + e_{hj}\right) - B_{ij} \sin\left(wt + e_{hj}\right)\right)$$

Equation 2

where j is the mode of oscillation and i is the direction of the load. The phase lag e_{hj} is defined as the phase lag between the forced motion of the cross section in still water and the velocity potential of the fluid. The radiated waves have an amplitude z_{ja} and y_{wl} is half the breadth of the section at the waterline. The potential coefficients A_{ij} and B_{ij} and the phase lags e_{hj}, expressed in conformal mapping coefficients, are given by Tasai (1960) and Tasai (1961) and used by Journée (1992b) in the computer code SEAWAY.

These loads can be expressed in terms of in and out phase components with the harmonic oscillations:

$$X_{hi} = \frac{r \cdot a_{ij}}{w^2 \cdot x_{ja}} \cdot \left(\frac{g z_{ja}}{p} \right)^2 \cdot \cdot \begin{pmatrix} \left(A_{ij} Q_{0j} + B_{ij} P_{0j}\right) \cdot \cos wt \\ + \\ \left(A_{ij} P_{0j} - B_{ij} Q_{0j}\right) \cdot \sin wt \end{pmatrix}$$

Equation 3

with $a_{22} = 2$, $a_{24} = 4 / y_{wl}$, $a_{33} = 2$, $a_{44} = 8$, $a_{42} = 4 y_{wl}$ and for the terms P_{0j} and Q_{0j}:

$$P_{0j} = -\frac{x_{ja}}{z_{ja}} p \frac{w^2}{g} y_{wl} \cdot \sin e_{hj}$$

$$Q_{0j} = +\frac{x_{ja}}{z_{ja}} p \frac{w^2}{g} y_{wl} \cdot \cos e_{hj}$$

Equation 4

The phase lag e_{hj} between the velocity potentials of the fluid and the forced motions of the cross section in still water is now incorporated in the coefficients P_{0j} and Q_{0j}, and can be obtained by using:

$$e_{hj} = \arctan\left(\frac{-P_{0j}}{+Q_{0j}} \right)$$

Equation 5

Equation 5 will be used further on to obtain the wave load phases.
Generally, these hydrodynamic loads are expressed in potential mass and damping terms:

$$\begin{aligned} X_{hi} &= -M_{ij} \cdot \ddot{x}_j - N_{ij} \cdot \dot{x}_j \\ &= M_{ij} w^2 x_{ja} \cdot \cos wt + -N_{ij} w x_{ja} \cdot \sin wt \end{aligned}$$

Equation 6

with:

$$M_{ij} = rb_{ij} \frac{A_{ij}Q_{0j} + B_{ij}P_{0j}}{P_{0j}^2 + Q_{0j}^2}$$

$$N_{ij} = rb_{ij} \frac{A_{ij}P_{0j} - B_{ij}Q_{0j}}{P_{0j}^2 + Q_{0j}^2} \cdot w$$

Equation 7

where: $b_{22} = 2y_{wl}^2$, $b_{24} = y_{wl}^3$, $b_{33} = 2y_{wl}^2$, $b_{44} = 2y_{wl}^4$ and $b_{42} = 4y_{wl}^3$.

Note that in the general notation of Equation 6 the phase lag information e_{hj} is lost. This, however, is not the case when Equation 3 is used.

Tasai (1965) has used the following potential damping coefficients in his formulations of the hydrodynamic loads:

$$N_{42} = \frac{N_{44}}{l_w} \qquad N_{24} = N_{22} \cdot l_w$$

Equation 8

in which l_w is the lever of the rolling moment. Because the values of these two coefficients are equal, the potential roll damping coefficient N_{44} can be obtained from the sway damping coefficients N_{22} and the coupling coefficients N_{24} or N_{42} by:

$$N_{44} = \frac{N_{24}^2}{N_{22}} = \frac{N_{42}^2}{N_{22}}$$

Equation 9

Computational validation of these relations for the roll damping coefficients has been carried out with the strip-theory program SEAWAY of Journée (1992a) for all cross sections of the 200,000 TDW tanker.

Figure 2 shows an example of these validations for two cross sections of this ship. The figure shows a perfect agreement, as has been found for all sections.

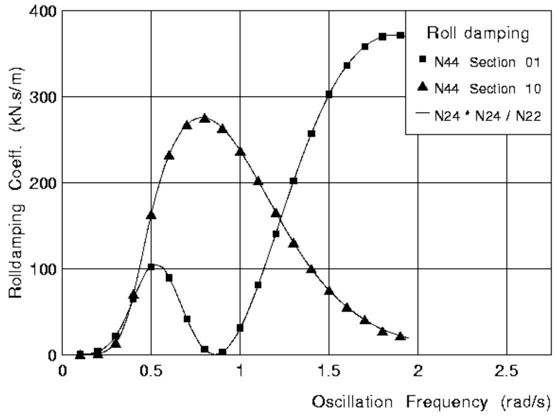

Figure 2 Example of Roll Damping Coefficients for Two Cross Sections of a 200,000 TDW Crude Oil Carrier

From Equation 9 follows that the potential damping coupling coefficients $N_{24} (= N_{42})$ can be obtained from the potential sway and roll damping coefficients N_{22} and N_{44} by:

$$|N_{24}| = |N_{42}| = \sqrt{N_{22} \cdot N_{44}}$$

Equation 10

The sign of these coupling coefficients follows from the course of N_{44} with frequency. This relation (Equation 10) will be used further on, to obtain the wave loads for roll.

4 Wave Loads

Consider a fixed infinite long cylinder in regular beam waves with wave frequency w and small wave amplitude z_a. The 2-D wave elevation at the cross section in beam waves is defined by:

$$z = z_a \cdot \cos(wt - kx_{2b})$$

Equation 11

The wave loads X_{wi} for sway, heave and roll respectively, on a cross section consist of a Froude-Krilov part F_{wi} and a diffraction part R_{wi}:

$$X_{wi} = F_{wi} + R_{wi} \qquad \text{for: } i = 2, 3, 4$$

Equation 12

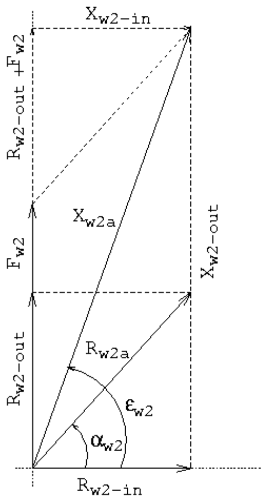

Figure 3 Vector Diagram of Wave Force Components for Sway

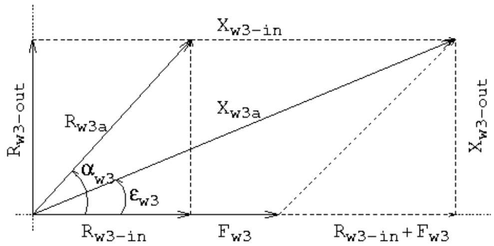

Figure 4 Vector Diagram of Wave Force Components for Heave

Figure 3 and Figure 4 show the in and out phase parts of the wave load components for sway and heave. The roll moment vectors are similar to the sway force vectors.

Note that the sign conventions for the loads are:

$$\begin{aligned} X &= \Re\left\{X_a \cdot e^{i(wt+e)}\right\} = \Re\left\{\left(X_{in} + iX_{out}\right) \cdot e^{iwt}\right\} \\ &= X_a \cdot \cos(wt + e) \\ &= X_a \cdot \cos e \cos wt - X_a \cdot \sin e \sin wt \\ &= X_{in} \cdot \cos wt - X_{out} \cdot \sin wt \end{aligned}$$

The Froude-Krilov load F_{wi} can be calculated by a simple integration of the water pressure on the cross section, where it is assumed that the cross section is fixed and not disturbing the incoming waves.

Methods to determine the diffraction loads R_{wi} are discussed in the following paragraphs.

4.1 Relative Motion Approach

In the relative motion approach in the strip theory, see for instance Vugts (1970), sectional averaged orbital motions of the water particles in the sway and heave direction are defined to obtain the diffraction loads in these directions.

The sectional averaged orbital motions in the sway and heave direction in the origin are respectively:

$$z_2^* = z_{2a}^* \cdot \sin wt$$
$$z_3^* = z_{3a}^* \cdot \cos wt$$

Equation 13

The (reduced) wave amplitudes for sway z_{2a}^* and heave z_{3a}^* are determined from the pressure distribution along the contour of the cross section in the undisturbed wave, see Journée (1992b), by:

$$z_{2a}^* = \frac{\displaystyle\int_{-T}^{0} \frac{\sin\left(-kx_{2b}\sin m\right)}{-kx_{2b}\sin m} e^{kx_{3b}} x_{2b}\, dx_{3b}}{\displaystyle\int_{-T}^{0} x_{2b}\, dx_{3b}} \cdot z_a$$

$$\begin{aligned} z_{3a}^* = \Bigg\{ & y_{wl} \frac{\sin\left(-ky_{wl}\sin m\right)}{-ky_{wl}\sin m} - \\ & k \int_{-T}^{0} \frac{\sin\left(-kx_{2b}\sin m\right)}{-kx_{2b}\sin m} e^{kx_{3b}} x_{2b}\, dx_{3b} \Bigg\} \cdot z_a \end{aligned}$$

Equation 14

The depth of the cross section is T and y_{wl} is half the breadth of the waterline.

The diffraction parts of the wave loads are determined from the potential mass and damping coefficients and the sectional averaged orbital velocity and acceleration of the water particles, using:

$$R_{w2} = M_{22} \cdot \ddot{z}_2^* + N_{22} \cdot \dot{z}_2^*$$
$$R_{w3} = M_{33} \cdot \ddot{z}_3^* + N_{33} \cdot \dot{z}_3^*$$
$$R_{w4} = M_{24} \cdot \ddot{z}_2^* + N_{24} \cdot \dot{z}_2^*$$

Equation 15

Note that in the sway force and the roll moment terms with $\ddot{z}_4^*$ and $\dot{z}_4^*$ will not appear since the fluid is assumed to be free of rotation.
Equation 15 can be written as:

$$R_{wi} = R_{wia} \cdot \cos(wt + a_{wi}) \qquad i = 2, 3, 4$$

Equation 16

where the amplitudes of the diffraction wave loads are:

$$R_{w2a} = w\sqrt{(wM_{22})^2 + (N_{22})^2}$$
$$R_{w3a} = w\sqrt{(wM_{33})^2 + (N_{33})^2}$$
$$R_{w4a} = w\sqrt{(wM_{24})^2 + (N_{24})^2}$$

Equation 17

and where the phases of the diffraction loads are calculated with:

$$a_{w2} = \arctan\left(\frac{w \cdot M_{22}}{N_{22}}\right)$$
$$a_{w3} = \arctan\left(\frac{N_{33}}{-w \cdot M_{33}}\right)$$
$$a_{w4} = \arctan\left(\frac{w \cdot M_{24}}{N_{24}}\right)$$

Equation 18

Originally, this relative motion approach was introduced to obtain the heave and pitch motions in bow waves and this showed a fair agreement with results obtained when using the diffraction theory.

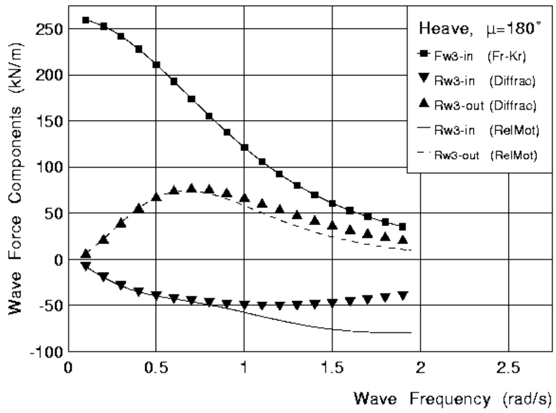

Figure 5 Calculated Vertical Wave Loads on Cross Section 01 in Head Waves

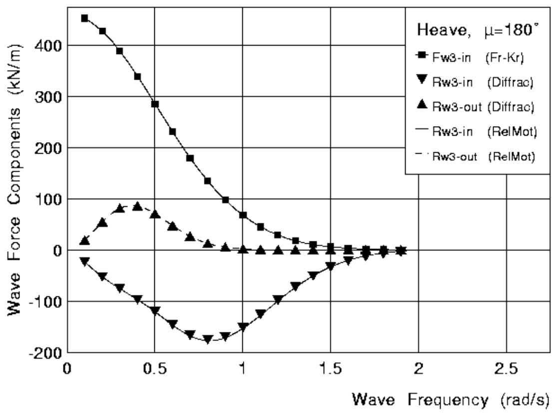

Figure 6 Calculated Vertical Wave Loads on Cross Section 10 in Head Waves

Figure 5 and Figure 6 respectively show the Froude-Krilov force (Fr-Kr) and a comparison of the results of the relative motion approach (RelMot) with those of the diffraction theory (Diffrac) for wave load components for heave in head waves of the two cross sections.
The figures show a fair agreement between the two theories. A perfect fit will be obtained for cross sections with vertical side walls, such as section 10. In case of heeled side walls, such as section 01, deviations will be found in the diffraction part of the wave loads at higher frequencies.
However, very poor agreements between the two theories can be found in beam waves, especially in the higher frequency region of the wave loads as will be shown later on in the paper.

4.2 Radiated Wave Approach

The disadvantage of using direct diffraction theory calculations (Diffrac) is that the strip-theory program is slowed down since the diffraction problem has to be solved for each wave direction, while in the relative motion theory the diffraction force amplitude and phasing are calculated directly with the previously obtained hydrodynamic coefficients.

Therefore a simple calculation method to obtain the wave loads, based on the diffraction theory but without solving the diffraction problem itself for each wave direction, is obviously welcome. As a first attempt a method will be presented here for beam waves only, to obtain the wave loads using only results of the hydrodynamic potential coefficient calculations.

The overall wave loads for sway, heave and roll are defined by:

$$X_{w2} = X_{w2a} \cdot \cos(wt + e_{w2})$$
$$X_{w3} = X_{w3a} \cdot \cos(wt + e_{w3})$$
$$X_{w4} = X_{w4a} \cdot \cos(wt + e_{w4})$$

Equation 19

4.2.1 Wave Load Amplitudes

Consider a cross section of an **oscillating body**, moving with zero forward speed in the direction j with frequency w and amplitude x_{ja} in still water. The energy required for this oscillation (left hand side of the equation of motion), should be equal to the energy radiated from this cross section by the damping waves.

So:

$$\frac{1}{T_{osc}} \int_0^{T_{osc}} N_{jj} \dot{x}_j \cdot \dot{x}_j \, dt = \frac{rgz_a^2}{2} \qquad j = 2, 3, 4$$

Equation 20

in which:

$c = g / w$	Wave velocity
g	Acceleration of gravity
$T_{osc} = 2p / w$	Period of oscillation
$k = 2p / l$	Wave number
l	Wave length
N_{jj}	2-D potential damping coeffi-cient
$x_j = x_{ja} \cos wt$	Harmonic displacement in direction j
z_a	Amplitude of radiated waves
r	Density of water
t	Time

From substitution of Equation 1 into Equation 20 it follows that the ratio of the motion amplitude of the body x_{ja} and the radiated wave amplitude z_a can be written in terms of the 2-D damping coefficient N_{jj} as follows:

$$\frac{x_{ja}}{z_a} = \frac{1}{w} \cdot \sqrt{\frac{rgc}{N_{jj}}} \qquad j = 2, 3, 4$$

Equation 21

The motion amplitude x_{ja} also represents a sectional-averaged amplitude of the harmonic orbital motions of the water particles, relative to the oscillating cross section.

Consider now a cross section of a **fixed body** with zero forward speed, subject to regular incoming beam waves (right hand side of the equation of motion), having an energy equal to Equation 20.

Then the incoming wave has an amplitude z_a and the sway and heave motion amplitudes x_{ja} from Equation 21 now represents sectional averaged harmonic orbital motion amplitudes of the water particles relative to the fixed cross section. Because of the similar energies (left and right hand side of the equation of motion) these motion amplitudes are equal. Note

that the orbital motion x_4 of the water particles does not exist here, since the fluid is assumed to be free of rotation.
The amplitude of the exciting wave force for sway and heave due to the radiated damping waves is given by:

$$X_{w2a} = N_{22} \cdot \dot{x}_{2a} = N_{22} \cdot w \cdot x_{2a}$$
$$X_{w3a} = N_{33} \cdot \dot{x}_{3a} = N_{33} \cdot w \cdot x_{3a}$$

Equation 22-ab

This leads to the following expression for the wave force amplitudes for sway and heave in beam waves:

$$\frac{X_{w2a}}{z_a} = \sqrt{rgc \cdot N_{22}}$$
$$\frac{X_{w3a}}{z_a} = \sqrt{rgc \cdot N_{33}}$$

Equation 23-ab

For an oscillating body in still water x_{4a} / z_a exists, but for the fixed body in waves it does not exist, because the fluid is free of rotation. The wave moment amplitude for roll in this fluid is defined by:

$$X_{w4a} = |N_{24}| \cdot \dot{x}_{2a} = |N_{24}| \cdot w \cdot x_{2a}$$

Equation 22-c

With Equation 10, Equation 22-a and Equation 23-a a similar relation as for the sway and heave forces will be found for the roll moments:

$$\frac{X_{w4a}}{z_a} = \sqrt{rgc \cdot N_{44}}$$

Equation 23-c

So the three wave load amplitudes are given by:

$$\frac{X_{w2a}^2}{N_{22}} = \frac{X_{w3a}^2}{N_{33}} = \frac{X_{w4a}^2}{N_{44}} = rgc \cdot z_a^2$$

Equation 24

The relation in Equation 24 between the exciting wave loads and the amplitude of the radiated waves at infinity was already shown by Newman (1962), who derived this by using Green's second identity. This method to obtain the exciting wave amplitude from the potential damping coefficient is known as the Haskind-Newman method.

4.2.2 Wave Load Phases

The phase lags e_{h2} and e_{h3} are the phases between the velocity potentials of the fluid and the translations of the *oscillating body* relative to the water particles in still water. To obtain the phase lags e_{w2} and e_{w3} between the velocity potentials of the fluid and the translations of the *fixed body* relative to the water particles in beam waves, the phase lags e_{h2} and e_{h3} have to be diminished by p. Because of reference of sway to the vertical wave motions, $p/2$ has to be added to e_{w2}.

$$e_{w2} = e_{h2} - p + \frac{p}{2} = e_{h2} - \frac{p}{2}$$
$$e_{w3} = e_{h3} - p$$

Equation 25-ab

In waves the body is fixed and the fluid is assumed to be free of rotation, so no relative roll motions of the water particles are present. This means that for the determination of the phase lag e_{w4} no use can be made of the phase e_{h4} between the velocity potential of the fluid and the rotation of the oscillating body relative to the water particles in still water.
Referring to Figure 2 and using Equation 8 and Equation 24 it can be shown that:

$$\left| \frac{F_{w4} + R_{w4-out}}{F_{w2} + R_{w2-out}} \right| = \left| \frac{R_{w4-in}}{R_{w2-in}} \right| = \left| \frac{N_{24}}{N_{22}} \right|$$

From analysing sway and roll data of cross sections with various shapes, calculated with the diffraction theory by the strip-theory program SEAWAY of Journée (1992a), it was found that the sign of these ratios follows from the sign of N_{24}:

$$\frac{F_{w4}+R_{w4-out}}{F_{w2}+R_{w2-out}}=\frac{R_{w4-in}}{R_{w2-in}}=\frac{N_{24}}{N_{22}}$$

Equation 26

where R_{wi-in} denotes the in phase part and R_{wi-out} the out phase part of the diffraction loads.

From calculated diffraction theory data in bow and quartering waves, it appeared that the expression for the out phase part of the wave loads is valid for low frequencies only. The expression for the in phase part of the wave load is valid in the whole wave frequency range for all wave directions:

$$\frac{R_{w4-in}}{R_{w2-in}}=\frac{N_{24}}{N_{22}} \qquad (m=......)$$

The wave moment for roll must be obtained from the wave force for sway using Equation 26 and Equation 23-ab:

$$X_{w4}=\frac{N_{24}}{N_{22}}\cdot X_{w2}$$
$$=N_{24}\cdot\sqrt{\frac{rgc}{N_{22}}}\cdot X_{w2}$$

Equation 27

So the phase lag becomes:

$$e_{w4}=e_{w2}=e_{h2}-\frac{p}{2}$$

Equation 25-ab

When N_{24} becomes negative, X_{w4} will change sign and e_{w4} has to be increased by p.

5 Results of Calculations

From the foregoing it follows that in the radiated wave approach the wave loads are defined by:

$$X_{w2}=\sqrt{rgc\cdot N_{22}}\cdot z_a\cos(wt+e_{h2}-p/2)$$
$$X_{w3}=\sqrt{rgc\cdot N_{33}}\cdot z_a\cos(wt+e_{h3}-p)$$
$$X_{w4}=N_{24}\cdot\sqrt{\frac{rgc}{N_{22}}}\cdot z_a\cos(wt+e_{h2}-p/2)$$

Equation 28

The wave loads in beam waves can be obtained using the results of the potential mass and damping calculations where the phases e_{hi} are obtained from Equation 5.

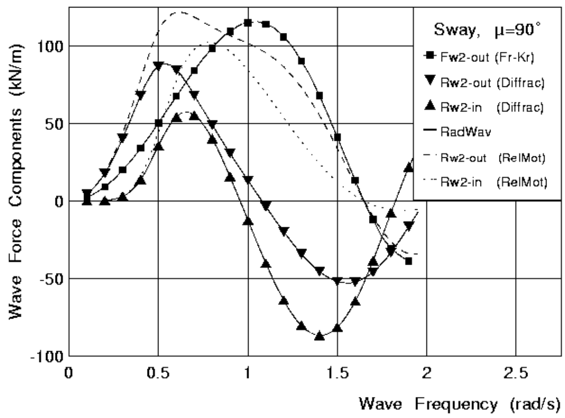

Figure 7 Wave Load Components of Sway for Cross Section 01 in Beam Waves

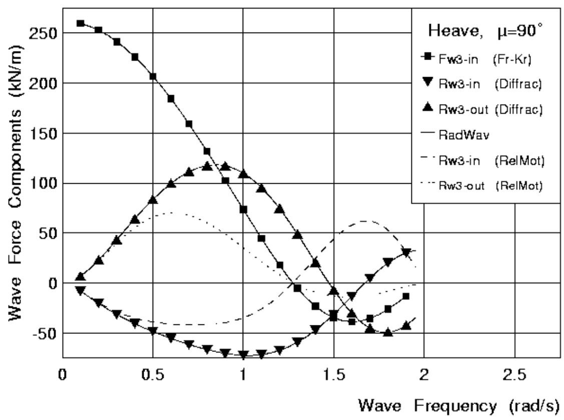

Figure 8 Wave Load Components of Heave for Cross Section 01 in Beam Waves

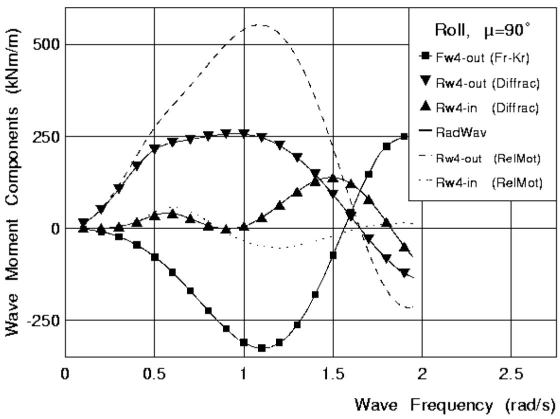

Figure 9 Wave Load Components for Roll on Cross Section 01 in Beam Waves

Figure 7, Figure 8 and Figure 9 show all calculated wave load components in beam waves for cross section 01. These figures show that the use of this radiated wave approach (RadWav) in beam waves, with information on the potential coefficients only, leads to a perfect agreement with the results of the diffraction theory (Diffrac). The results of the relative motion approach (RelMot) fit the results of the diffraction theory (Diffrac) at very low frequencies only. This was found for all cross sections of the 200,000 TDW tanker.

From Figure 3 and Figure 4 it can be seen that the in and out phase parts of the diffraction parts of the wave forces for sway and heave can be obtained from the Froude-Krilov force F_{wi} and the overall wave force amplitude X_{wi}, when the phase lag a_{wi} or the phase lag e_{wi} is known.

From analysing phase lag a_{wi} data, obtained by diffraction theory calculations, it appeared that for low frequencies a_{wi} tends to the definition given in Equation 18, as used in the relative motion approach. This is shown in Figure 10 for sway and in Figure 11 for heave. From analysing phase lag e_{wi} data, obtained from potential coefficient or diffraction theory calculations, it appeared that in the higher frequency region these phase lags can be approximated by:

$$e_{w2} \approx e_{w3} \approx k \cdot y_{wl} \qquad \text{for } w \to \infty$$

Equation 29

This is shown in Figure 10 and Figure 11 too. For cross sections with a vertical side wall, such as section 10, this approximation is almost perfect.

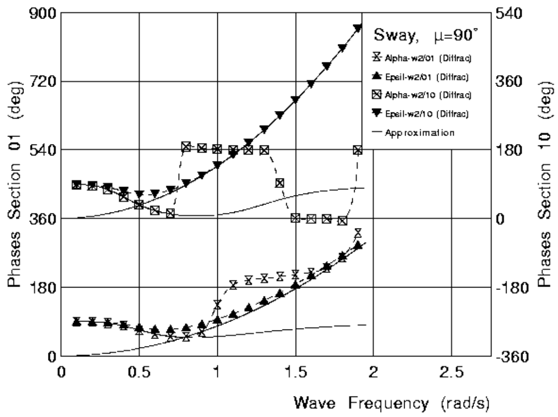

Figure 10 Phase Lags of Sway Wave Forces for Cross Sections 01 and 10 in Beam Waves

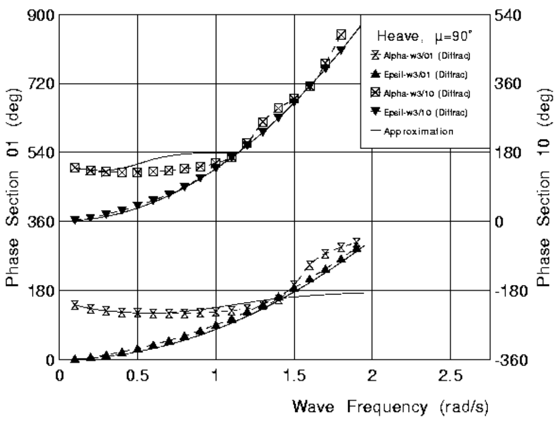

Figure 11 Phase Lags of Heave Wave Forces for Cross Sections 01 and 10 in Beam Waves

6 Dynamic Swell-Up

Let an oscillating body produce damping waves, Δz, with amplitude Δz_a. When the vertical relative motions of a point fixed to the vessel with respect to the wave elevation are calculated, the influence of the radiated damping waves must be added to the undisturbed incoming wave. Then the vertical relative motions s_3 in a point (x_{1b}, x_{2b}) can be calculated using:

$$s_3(x_{1b}, x_{2b}) = z(x_{1b}, x_{2b}) + \Delta z - x_3(x_{1b}, x_{2b})$$

Equation 30

where z is the elevation of the incoming waves and $x_3(x_{1b}, x_{2b})$ is the vertical motion of the vessel in (x_{1b}, x_{2b}) and where the elevation of the radiated waves due to the heaving cross section (dynamic swell-up) is given by:

$$\Delta z = \Delta z_a \cdot \cos(wt - k|x_{2b}| - p)$$

Equation 31

For zero forward speed, the ratio of the amplitude of the local motion x_{3a} at cross section x_{1b} of the body and the amplitude of the produced transverse radiated waves Δz_a follows from Equation 21:

$$\frac{x_{3a}(x_{1b})}{\Delta z_a} = \frac{1}{w} \cdot \sqrt{\frac{rgc}{N_{33}}}$$

Using this equation, the ratio of the amplitudes of the radiated waves due to the heaving cross section and the incoming wave amplitude becomes:

$$\frac{\Delta z_a}{z_a} = \frac{x_{3a}(x_{1b})}{z_a} \cdot w \cdot \sqrt{\frac{N_{33}}{rgc}}$$

Equation 32

An experimental determination of the vertical relative motions of a model in a towing tank at zero forward speed is hardly impossible, because of tank wall interference. A forward speed is required to obtain reliable experimental data. Therefore Equation 32, valid for zero forward speed, is extended with forward speed effects in a simple manner.

In case of a forward ship speed V, the section is oscillating with the encounter frequency w_e and therefore the wave number and the wave velocity of the radiated waves have to be based on the encounter frequency as well.

$$k_e = \frac{w_e^2}{g} \quad \text{and} \quad c_e = \frac{g}{w_e}$$

Due to the forward speed the radiated waves are swept back in the wake. The wave elevation at a certain point (x_{1P}, x_{2P}) in the vessel fixed reference system is now a result of the radiated waves from a more forwardly located cross section. The x_{1b}-position of this cross section can be simply calculated using:

$$x_{1b} = x_{1P} + |x_{2P}| \cdot \frac{V}{c_e}$$

Equation 33

In case of a forward ship speed, the amplitude of the radiated waves can now be calculated using the damping coefficient based on the encounter frequency and Equation 33 for x_{1b}:

$$\frac{\Delta z_a}{z_a} = \frac{x_{3a}(x_{1b})}{z_a} \cdot w_e \cdot \sqrt{\frac{N_{33}(x_{1b})}{r g_e c}}$$

Equation 34

This calculation method has been verified with some results of experiments in regular head waves, carried out in the past by Journée (1976), with a self-propelled 1:50 model of a fast cargo ship. In the ballast condition of the model, the vertical relative motions have been measured at four Froude numbers (0.15, 0.20, 0.25 and 0.30) at a section placed at 10% of the vessel's length aft of the forward perpendicular.

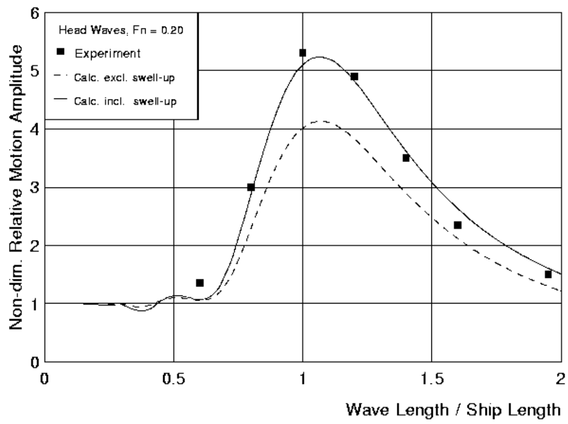

Figure 12 Vertical Relative Motions Forward of a Fast Cargo Ship at $Fn = 0.20$

Figure 12 shows an example of the results for $Fn = 0.20$ in head waves. The vertical ship motions at forward speed have been calculated with the well-known ordinary strip-theory method. The measured relative motions are compared with calculated motions with and without the effect of the dynamic swell-up. Similar agreements as presented here, have been found for the other three Froude numbers. When accounting for the dynamic swell-up, the calculations show a strongly improved agreement between the measured and the predicted relative motions, especially at resonance frequencies.

7 Conclusions

The roll-roll damping coefficient can be obtained from the sway-sway and the sway-roll potential damping coefficients, in a very simple manner.
When comparing the computational results of the relative motion approach (RelMot) with those of the diffraction theory (Diffrac), fair agreements have been found in head waves. In beam waves however, large discrepancies are found.
The computational results of the radiated wave approach (RadWav) show perfect agreement with results of the diffraction theory (Diffrac).

The addition of a simple method to determine the dynamic swell-up of the waves in the calculation of the vertical relative motions forward results into a fair prediction of these motions in regular head waves.

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